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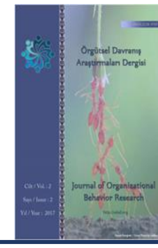

THE ROLE OF SCHOENFELD’S CONTROL FACTOR IN REDUCING THE TENTH GRADE MATH STUDENTS' MISCONCEPTIONS IN GEOMETRY

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ABSTRACT

By enhancing control abilities, the problem solver can use most of his resources to solve difficult problems with greater efficiency, and in the case of lack of control abilities, his knowledge resources may be wasted or not utilized. The present study aims to investigate the role of Schoenfeld’s control factor in reducing the 10th grade math students’ misconceptions in geometry. This study is an applied, quasi-experimental research. The statistical population it includes all 10th grade math students in Baneh city in the academic year of 2018-2019, 40 of whom were selected as sample. Data were collected utilizing two standard researcher-made tests used in a pretest-posttest design. The validity of the questionnaires was examined by sixteen experienced mathematics teachers. Cronbach's alpha was used to confirm the reliability of the questions. The results showed that the control factor can play a role in reducing students' misconceptions in geometric concepts and problems.

Keywords: *Control, Misconception, Learning, Geometry.*

INTRODUCTION

Geometry has had a major contribution to mathematics even in ancient times, and philosophers believed in the worthiness of knowing it as long as Plato inscribed the phrase "Let None But Geometers Enter Here." above the entrance to his Academy (Sibley, 1998). This famous phrase indicates the importance of geometry and its application to other sciences, such as philosophy and logic. Also, as a tool for understanding, describing, and interacting with the space in which we live, geometry teaching has a special place and should be considered.

Two goals are basically sought in learning geometry: the development of thinking skills and the formation of spatial intuition. Spatial intuition refers to how an individual’s view on space and areas in the real world is (Alex and Mammen, 2016). One of the basic goals of teaching mathematics is to improve students' geometric thinking. Geometric thinking is important in many scientific, technological, and professional issues (Tahani, 2016). On the other hand, if mathematics teachers are not well prepared to teach geometry, they would influence the students’ knowledge base and real-world situations (Reyhani, 2010).

Although geometry is one of the most intuitive and tangible parts of mathematics, according to the experiences of teachers and students, as well as mathematics education research, teaching and learning geometry has faced many problems, so that many teachers and students have no interest in geometry (Mansouri, 2009).

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Wherever there are education and learning, inadequate and inappropriate learning in some materials and concepts is not unexpected, so some misconceptions and disabilities take place due to them. Misconceptions in mathematics, for a variety of reasons, as well as in various forms, arise from both teachers and students, ranging from minor problems and ambiguities to widespread and significant disabilities (Alam al-Hodaii, 2009).

Identifying and discovering students' misconceptions is important to mathematics teachers because they can adjust their teaching method to some extent based on students' misconceptions. Detection of misconceptions will help us understand what methods, when and where are effective in student learning (Azarang, 2008).

On the other hand, as pointed out by Schoenfeld, control means to select and apply appropriate resources and strategies that help solve the problem. By enhancing control abilities, the problem solver can use most of his resources to solve difficult problems with greater efficiency, and in the case of lack of control abilities, his knowledge resources may be wasted or not utilized.

It seems that enhancing control skills as well as rooting up and analyzing errors can be greatly effective in solving students' problems related to learning geometry and making them more successful. In this study, it was tried to use control tools in teaching geometry to study their effectiveness in reducing students' misconceptions.

THEORETICAL FOUNDATIONS

Geometry

Since ancient times, geometry has played a prominent role in mathematics, and it was so important in ancient Greece so that an individual, who did not know geometry, was not allowed to join the circle of mathematicians. Plato, who taught philosophy in his school, attached great importance to geometry for the development of thought and reasoning, and like many scholars and philosophers, he found the study of geometry to be very useful and necessary for the study of philosophy as well as the development of thought and reasoning (Sharaf al-Din, 1998). Geometry is one of the most important mathematical fields, and provides experiences that help students develop their understanding of shapes and their properties, and enable them to relate geometrical problems to real-world problems (Sherard, 1981).

What matters is that we live in the real world and the real world is geometric. The dual nature of geometry, as a field of theory and practical experience, enables mathematics teachers to establish a link between theory and students' daily knowledge (Reyhani, 2005). For centuries, mathematics was treated as the supreme lesson for the development of reasoning power, and Vives, who lived in the fourth century, described mathematics as a lesson displaying the power of the mind (Najafi, 2000). Geometry is a branch of mathematics, that describes points, lines, maps, and spatial shapes, and the relations between these shapes describe the sizes of geometric shapes, such as length, angle, area, and volume (Yildiz, 2009).

Yee (2002) knows geometry to be a science for the study of space and the systematic ways of looking at space around man. According to him, in a school mathematics curriculum, teaching geometry aims at developing spatial intuition and understanding, enhancing rational thinking skills, and he defines it as a prerequisite for other mathematical sections. Reyhani (2005) has also emphasized that geometry has been developed to understand and interpret various phenomena, and thus, it is necessary to examine the geometric thinking needed to understand these phenomena and how they are developed.

Mathematics researchers and teachers have provided various definitions of geometry. One of these proper definitions presented by Felix Klein: “geometry is a space, together with a group of transition within it, and considering different groups of transitions, there are different geometries which form a hierarchal geometry”.

Reasoning and proof

Reasoning and proof are of skills that have a special place in everyday life in general and in mathematics education in particular. Mathematical proof is a logical sequence of arguments, and begins with a set of definite data (such as axioms, definitions, assumptions, and previous proven results) and reaches a valid conclusion using logical steps. Proof is a complex mathematical activity and the examination of its nature depends on many factors including cognitive, mathematical, historical, epistemological and social factors (Weber, 2005).

Ross (2000) believes that reasoning forms the basis of mathematics. "While science is confirmed by observation, mathematics is verified by logical reasoning” he states, “and if the capability of reasoning does not develop in students, for them, mathematics becomes a set of repeated procedures and examples without thinking about why they are." Schoenfeld (1994) argues that proving has the most conceptual errors in mathematics curriculum and we really need to classify it.

One of the major reasons for students' difficulties in conception, understanding and presentation of proof seems to be that some teachers do not consider what is considered by the students to be the reason and evidence for proving a proposition, and impose on them implicit proof methods and rules, instead of gradually correcting their understanding of proof and presentation of valid reasoning. It is important to note that in many cases, these rules are disproportionate and inconsistent with what convinces students, especially in preliminary courses. In general, for teachers and curriculum planners, it is essential to know what guides students' thinking about proof (Harel and Sowder, 1998).

Misconception

Misconception means a wrong idea or opinion resulting from a misunderstanding of something. Misconception usually occurs when, in particular, some idea is created in a student's mind and then, the student, in general, generalizes the idea incorrectly (Ibrahimi, 2016).

Mestre (1989) believes that some of the ideas that students use to construct the concepts of their world, may be imperfect and not all truth. He calls this defect “conceptual errors” (Karimikia, 2012). Michael (2002) defines the conceptual error as a mismatch between the concept we want students to learn and the mental model they construct in their minds. Research has made clear that errors occur mainly because students have difficulty in understanding the educational strategies employed by the teacher (Confrey, 1990).

Klammer (1998) divided the sources of conceptual errors into three categories (Kutluay, 2005):

- Experiences: For example, students observe that a stone falls faster than a feather. But these two bodies touch the floor at the same time in a vacuum if dropped from the same height. Here, the results of experiments and environmental experiences are inconsistent and cause conceptual errors.

- Language: Many similes or metaphors are rooted in language. Although metaphors help students better understand the world around them, they do not always work well in science.
- Curriculum: In the teaching process, teachers teach a simple form of subjects, and students cannot fully explore and develop the idea or theory in question rationally.

Identifying and discovering students' misconceptions is important to mathematics teachers because they can adjust their teaching method to some extent based on students' misconceptions. Detection of misconceptions will help us understand what methods, when and where are effective in student learning (Azarang, 2008).

In many cases, it is observed that the student wrote all the information needed to solve the problem in his/her exam sheet, but failed to properly sort it out, and finally, reached no final answer, or solved the problem using very long explanation that was not needed. Accordingly, the main source of these problems is the lack of skills, that Schoenfeld calls it "control" skill (Karimian, 2015).

If the student has the ability to use the "control" component, he would not become confused with different solutions while solving the problem and can make good use of his time. The approaches alone cannot guarantee the success of problem solving; rather, the key factors for successful solving of mathematical problems is that students can make correct control decisions in different situations (Faramarzpour, 2016).

In fact, "control" is related to the way people use the information available to them. Like how to solve a problem, what plan to follow, when to give up the solution, and at that time what solution to start. This type of decision-making is not a hierarchical and predetermined process during problem solving. Most decisions are made "at the scene" and depend on the situation. In fact, one may do a lot of parallel work when solving a problem, but he has to decide which one is better. There are many examples that illustrate how bad control results in failure and how good control can prevent major deviations in problem solving, or even act as a positive factor in obtaining a solution (Ghaffari, 2011).

Control from Schoenfeld's perspective

Schoenfeld became aware of the critical and influential factor in students' skills, which he called "control strategy", by developing problem solving in students. In the analysis by Schoenfeld, control strategies relate to executive decisions, such as generation of alternative activities, evaluation of solutions, evaluation of what one might be able to do, investigation of the approaches one use, assessment of what one make to develop a solution, and so on (Karimian, 2015). Schoenfeld (1985) studied the factors influencing math problem solving considering Polya's (1945) four-step problem solving model. In his view, these include knowledge resources, math problem-solving strategies, control, and problem solver's belief system. The study of preliminary results highlighted the roles of these factors, and especially the role of control as a determining factor.

As Schoenfeld points out, control means to select and employ the right resources and strategies helping solve the problem. Among the control abilities, the following can be mentioned:

- Problem solving outline
- Review and decision making

- Conscious metacognitive knowledge

Research on math problem solving shows that one's awareness of his mathematical knowledge and how to use it in the right situation, as well as his ability to review his performance when and after solving problems (metacognitive ability) have a direct impact on his success in solving math problems (Samadi, 2000).

Schoenfeld (1985) defines control in the problem-solving process as "general decisions about the selection and utilization of resources and strategies" and argues that control includes analysis, design, implementation, and review and evaluation of the solution, that they all interact together. He refers to them as the overall pattern of problem-solving strategy.

Mohseni (2018), in a study on the effect of Schoenfeld's control factor on students' errors based on the Newman model presented in learning trigonometry, showed that enhancing control skill defined by Schoenfeld among students significantly reduces the error in reading, comprehension and conversion stages, but no significant reduction is observed in the error of processing and writing steps. Behzadi (2015) studied the third-grade students' problems in correct conception of geometrical concepts (misconception) and the teaching method used to improve these misconceptions. In this study, it was attempted to study some of the learning problems in geometry and provide some solutions, such as enhancing students' power of visualization, and being more familiar with the 3D space, for teaching geometric concepts.

METHOD

This study a quasi-experimental research in which a pretest-posttest design was applied using two experimental and control groups. For this purpose, a researcher-made test was developed to identify students' misconceptions in geometry, its validity and reliability were assessed, and students' misconceptions were identified by this test. Then, the students with the most misconceptions were selected and divided into two experimental and control groups.

The statistical population of this study included all tenth grade math students in Baneh city, who were studying in the academic year 2018-19 (total N= 120 students). To select samples, convenience sampling method was used in this study. Twenty students with the most misconceptions were selected as the sample from each class.

Given that the number of math students in each school of Baneh City was not sufficient to select both groups from one school, it was decided to select the students from two schools that their 10th grade math students are approximately on the same level. One class was selected from each school (two classes in total), and then, they were randomly selected as the experimental and control group. Then, the experimental group received 6 sessions of control skills training and the control group was traditionally taught for 6 sessions. Finally, post-test was performed on both groups and their performances were compared.

Two researcher-made tests were used to collect the required data. The first test was performed to detect misconception and the second one was performed as a post-test to examine the impact of the independent variable. To examine the questions and the relationships between the research variables, Levene's test was used to examine the consistency of the variances and the Mann-Whitney U test was used to compare the two groups in performance, and determine the significance level.

Research Question: Is Schoenfeld's control factor effective in reducing the 10th grade students' misconceptions in Geometry?

Table 1. Classification of misconceptions

Class	Sub-class	Misconception code	Reference
Misconception in the properties of polygons	Misunderstanding of the definition of specific polygons (rhombus, square, etc.)	1	McCrone and Martin (2004)
	Inability to apply the properties of polygons to prove	2	McCrone and Martin (2004)
	Misconception in calculating the area and perimeter of polygons	3	Machaba (2016)
Misconception in proof and reasoning	Indiscrimination of the equality of the corresponding components (sides and angles)	4	McCrone and Martin (2004)
	Using proposition to give a reason for congruence	5	Clements and Batista (1992)
	Inability to make changes in shape or complete it	6	Clements and Batista (1992)
Misconception in proportion and similarity	Incorrect use of proportionality	7	Mahlabela (2012)
	Using the equality of sides rather than proportionality to prove similarity	8	Mahlabela (2012)
Misconception in the use of theorems	Inability to use the Pythagorean relationship	9	Clements and Batista (1992)
	Incorrect mathematical calculations and poor understanding of formulas	10	Luneta (2015)

To facilitate the comparison and interpretation of data, the frequencies and percentage of different misconceptions observed in both groups were listed in Table (2).

Table 2: Frequencies and percentage of different misconceptions observed in experimental and control groups in pretest

	Misconception code	1	2	3	4	5	6	7	8	9	10
Experimental group	Frequency	5	14	11	12	9	16	6	5	7	7
	Percentage	25	70	55	60	45	80	30	25	35	35
Control group	Frequency	4	15	8	13	8	17	5	5	6	6
	percentage	20	75	40	65	40	85	25	25	30	30

According to Table (2), it can be seen that in the pre-test, in the experimental group, the highest frequency (n=16) and percentage is related the topic of triangle congruence (and the section of changing the shape) (misconception # 6), followed by misconception # 2 (frequency of 13), which is about the topic of polygons (and the section of understanding the properties of polygons to prove the related materials).

Table 3: Frequencies and percentage of different misconceptions observed in experimental and control groups in posttest

	Misconception code	1	2	3	4	5	6	7	8	9	10
Experimental group	Frequency	2	7	3	4	2	5	0	2	3	4
	Percentage	10	35	15	20	10	25	0	10	15	20
Control group	Frequency	2	8	2	7	4	12	3	3	3	3
	percentage	10	40	10	35	20	60	15	15	15	15

According to Table (3), where the results of posttest are listed in, in the control group, the highest frequency (N=12) is related to misconception # 6, followed by misconception #2 (N=8) and the lowest frequency (N=3) is related to misconception #8.

Table 4: Percentage of reduction of misconceptions in the experimental and control groups

Misconception code	1	2	3	4	5	6	7	8	9	10
Percentage of reduction in the experimental group	60%	50%	73%	67%	78%	69%	100%	60%	57%	43%
Percentage of reduction in the control group	50%	47%	75%	46%	50%	29%	40%	40%	50%	50%

According to Table 4, it can be seen that the control factor had the greatest effect on misconception #7 by 100% reduction, followed by misconceptions #5 (78%) and #3 (73%). However, other misconceptions also decreased by at least 50%. But in control group, such a reduction was most evident in misconceptions related to computation. Moreover, the greatest reduction was observed in misconception #3, followed by misconception #8, and the least one was observed in misconception #6, which is associated with the understanding of geometric concepts.

Shapiro-Wik and Kolmogorov-Smirnov tests were used to test the normality of the data obtained from the experimental group in post-test.

Therefore, to compare the effect of separate trainings on each group, Mann-Whitney test (nonparametric t-test for independent groups) was used. In this test, H0 and H1 were defined as follows:

H0: Shoenfeld's control factor is not effective in reducing misconceptions.

H1: Shoenfeld's control factor is effective in reducing misconceptions.

Table 5: The significance level of the values of reduction in misconceptions in the experimental group

Indices \ Misconceptions	Misconception #1	Misconception #2	Misconception #3	Misconception #4	Misconception #5	Misconception #6	Misconception #7	Misconception #8	Misconception #9	Misconception #10
Mann-Whitney statistic	140.0	130.0	120.0	110.0	140.0	90.0	140.0	150.0	140.0	140.0
Sig.(two domains)	0.031	0.029	0.009	0.004	0.031	0.001	0.009	0.062	0.031	0.041
Sig. (one domain)	0.015	0.014	0.004	0.002	0.015	0.0005	0.004	0.031	0.015	0.021

Table 6: The significance level of the values of reduction in misconceptions in the control group

Indices \ Misconceptions	Misconception #1	Misconception #2	Misconception #3	Misconception #4	Misconception #5	Misconception #6	Misconception #7	Misconception #8	Misconception #9	Misconception #10
Mann-Whitney statistic	180.0	130.0	140.0	140.0	160.0	170.0	180.0	180.0	170.0	150
Sig.(two domains)	0.382	0.027	0.031	0.61	0.173	0.262	0.435	0.298	0.262	0.08
Sig. (one domain)	0.19	0.014	0.016	0.31	0.081	0.131	0.22	0.15	0.131	0.04

Table 7: The significance level of reduction of all misconceptions in the experimental group

Indices \ Misconceptions	Total
Mann-Whitney statistic	51.500
Sig.(two domains)	0.000
Sig. (one domain)	0.000

According to Table (7), designed for all the misconceptions in the experimental group, the significance of the Mann-Whitney test is obtained 0.00 and less than 0.05, so H0 is rejected, and it is concluded that the control factor is effective in reducing misconceptions.

CONCLUSION

The main purpose of this study was to determine the role of Shoenfeld's control factor in reducing the 10th grade math students' misconceptions in geometry. The data, required for investigating hypothesis and research question, were obtained using pretest-posttest design and comparing results. The results are investigated in following. According to the total value of misconceptions in the experimental group, the significance of the Mann-Whitney test was 0.00, showing that at the significance level of 0.05, the control factor was effective in reducing the misconceptions. So, totally, it can be said that control factor reduces students' misconceptions in geometry.

According to the results obtained in the control group, the significance levels of the values of reduction in misconceptions # 4, 5, 6, 9 and 10 are equal to 0.31, 0.081, 0.131, 0.131 and 0.04, respectively. That is, at the significant level of 0.05, for all the misconceptions, except for misconception #10, related to the research question, null hypothesis (H0: traditional education is not effective in reducing students' misconceptions) is accepted, indicating that traditional teaching method is not effective in reducing students' misconceptions.
The results obtained in posttest showed that the percentage of reduction in misconceptions # 7, 8 and 9, except for misconception #10, in the experimental group was greater compared to the control group, showing that control factor-based teaching outperforms the traditional teaching method in reducing all misconceptions, except for misconception #10, about which the better result was obtained by the traditional teaching method. The significance of this reduction, according to the results of Mann-Whitney test, is a reason to confirm the proper performance of the control factor-based teaching method. According to the results of Mann-Whitney test, the significance levels of the reduction values for misconceptions # 7, 8 and 10 in the experimental group were 0.004, 0.031 and 0.021, respectively, which means that at the significant level of 0.05 (5%), in the experimental group, for all misconceptions related to the research question, the null hypothesis (H0: Shoenfeld's control factor is not effective in reducing misconceptions) is rejected.
About misconceptions # 1, 2, 3 and 10, the results obtained for the experimental and control groups in posttest showed that the control factor-based teaching method outperforms the traditional teaching method in reducing misconceptions # 1 and 2, and the traditional teaching method outperforms control factor-based teaching method in reducing misconceptions #3 and 10. The significance of this reduction, according to the results of Mann-Whitney test, is a reason to confirm the proper performance of the control factor-based teaching method. So, about the third question, it can be claimed that in cases where students make mistakes in math calculations, the traditional teaching method has a positive performance and play a role in reducing students' misconceptions.
Based on the results of this study on the role of Shoenfeld's control factor in reducing students' misconceptions, it is suggested to mathematics teachers to apply control factor-based teaching method, instead of traditional teaching method, to reduce the errors of students and to increase their scientific and practical level, especially in geometry.

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