

MATHEMATICAL MODELS FOR RE-SCHEDULING FLIGHT IN CANCELLATION, DELAY AND DISPLACEMENT OF FLIGHTS

Seyed Zaman HOSSEINI, Alireza RASHIDI KOMIJAN*, Hassan JAVANSHIR

Department of Industrial Engineering, Roudehen Branch, Islamic Azad University, Roudehen, Iran.

*Corresponding Author:

E_mail: Alireza_Rashidi@yahoo.com

ABSTRACT

One of the basic planning problems of airlines is the results from the emergence of conditions such as poor weather conditions, flight crew delays and unpredictable technical problems. The occurrence of unexpected events has a huge impact on this predetermined planning. This paper attempts to run the flight network with a smaller number of aircraft by moving the hub airport, changing the time of some flights, removing a flight, and adding non-passenger flights. To implement this change, the mathematical model is presented and solved with the GAMS software. Since the proposed model and the type of aircraft routing issues are NP-Hard issues, a genetic algorithm was used, and a separate program was developed and executed in MATLAB to solve it, and it was shown that these algorithms can give acceptable (and possibly optimal) solutions in time, and therefore can be used to solve larger-dimensional problems. The results show that with a slight change in the flight network, one can save on the number of planes needed and plan the flight with fewer aircraft, which can help reduce costs and increase the profitability of airline companies.

Keywords: Flight Scheduling, Hub Airport, Fleet Shortage, Mathematical Model, Genetic Algorithm.

INTRODUCTION

Since our country has been faced with unjust sanctions by arrogant and colonial countries for many years, it has faced a shortage of fleet of aircraft, and the burnout of this fleet has caused many problems for airline companies and, of course, the people. In such a situation, the most important goal for airline companies is to increase the percentage of exploitation of this fleet and, given its wear and tear, their maintenance and repair services should be carried out in accordance with standardized plans in order to prevent any problems. Accordingly, airline companies try to schedule flights so that a single aircraft can carry out several successive flights, up to the maximum permissible daily flight, and make the most of the available fleet. The question now is how to achieve maximum efficiency with the current passenger fleet (lack of fleet and its weariness)? It may be possible to propose a better plan by rescheduling flights. The research seeks to increase the percentage of fleet utilization. This is possible by increasing the daily flight times of each aircraft to the permissible limit and, of course, requiring a robust planning and scheduling. By identifying the bottlenecks of each scheduling program and fixing it, the percentage of productivity is also rising. For example, traffic at a particular airport can cause unplanned delays; by changing the time of some flights, the traffic of that time period is reduced and delays could be prevented. Another way to improve performance is to add a minimum number of empty flights, which will increase the fleet utilization rate and reduce the number of aircraft needed. This research is limited to the third stage, routing and maintenance,

and does not deal with the other three. In fact, the input of the model presented in this study is an information file that includes the origin, destination, start time of the flight, the time of landing in the destination and the length of the flight, as well as the type of fleet and available number of them. The models presented in this study are attempting to check the current state of flight and identify bottlenecks. For example, the output of a model might be that by adding a non-passenger flight, the total number of flights can be made with a smaller aircraft, and this can save a lot of costs. Regarding the above, one can say that the research does not plan from the beginning, but by getting the current schedule and applying little changes, seeks to optimize it and increase performance indicators. The mathematical models presented in this study can be implemented with different objective functions and improve various indicators, such as increasing the fleet utilization rate, reducing flight costs, increasing maintenance opportunities at the airport and reducing the number of aircraft needed (Shafa'i & Jalayer, 2010).

RESEARCH BACKGROUND

Jarrah, et al. (1993), presented a model for rescheduling flights, attempting to minimize the overall cost of aircraft shortages. In this research, two separate models for flight have been used, which include the delay model and the removal model. In the delay model, flight delay and aircraft change can be accepted, and in the removal flight, removal and changing the aircraft, is considered as problem solving options. To solve the delay model in this case, a network flow model with the lowest cost is used (Jarrah et al., 1993). Yan and Yang conducted a research in 2005, which assumed that the disruption was due to the break of only one aircraft, but attempts have been made to consider the delay, removal, and transfer options of the aircraft to address the existing impairment. The emphasis of this research is to re-plan the flight so that the conditions could be restored as soon as possible. In this study, Yan and Yang considered four models in which three types of decisions, including the removal, delay and transfer of additional aircraft from one airport to another airport is proposed to resolve the disruption. The proposed solving methods for the first two flows in the network and for the last two models are the Lagrange liberation method. They used information from one of the Taiwanese airlines to evaluate the performance of their models (Yan & Yang, 2005). In another study, Letowski et al., (1998) planned crew and flight planning and the allocation of fleet after a disruption at the same time. The mathematical model used was a complex programming of very large integers, and its computational results in small examples cannot be achieved except in very short time. Hence, the researchers used the parsing method to solve the problem. The methods used in the study of these researchers to eliminate the disruption include delays and removal of flight; therefore, the issues of crew planning and fleet allocation are solved individually and applied to the main problem (Letovsky et al., 1998). Another study, Arjlo, et al. (1997) addressed the issue of rescheduling after a disruption caused by temporary disruption or delay. They considered the time period of one day to cover the disruption. The objective function of the proposed model is to minimize operating costs and maximize corporate earnings (Arjlo et al., 1997).

Sheikholeslami, et al. (2013) concluded that flights can be operated without disturbing the network. In addition, the increase in costs could be kept at an acceptable level. The base cost for comparison with the steady model is obtained by solving the typical problem of crew grouping. To cover the extraordinary flights it is inevitable to move the crew between stations, which has also been used in this study as well. Using the available resources, a mathematical model is



constructed, which is based on the rule of separate set issues. Initiative algorithms are used to create all possible working groups, and the original optimization problem has been solved with the GAMS software (Sheikholeslami et al., 2013). Rahimian, in his dissertation, presented a model for flight re-scheduling and made a comparison among the models of Anderson and Warbrand and his proposed model. The notable point in this study is that in the use of the prohibition algorithm, the neighborhood of each solution is determined in such a way that without the need to produce a new flight chain for each aircraft, by moving the transferable flights between the planes, each aircraft experiences different flight chains, to find the best solution and the chain is found for all planes. To find the neighborhoods, the limitation of the flow rate at each station and the amount of delay are considered. Other limitations are fined in the target function (Rahimian, 2003).

RESEARCH PROCEDURE

In this research, the basic mathematical model plus four new mathematical models will be presented for rescheduling and optimizing flight schedules. Given that the presented models are all NP-hard types, at last, a meta-initiative method is also proposed for solving these models (Genetic Algorithm). MATLAB software was used to implement the genetic algorithm.

Hypotheses

Hypotheses intended to solve the mathematical model for scheduling flights are:

- Input data is certain and definite.
- Only the flight plan of an airline is considered.
- The fleet of aircraft and their specifications, including number, capacity and ... is known.
- Information on flight routes including distance and flight time is known.

The mathematical model to reduce the number of aircraft needed

The parameters and indexes of the mathematical model for solving this problem are in accordance with Table 1.

Table 1: Parameters and indexes of the mathematical model

Index	Description	Index	Description
F	Major flights	R_j	Routes covering flight i
F'	Shifted flights	T	Time period
F_i	Major and shifted flights related to flight i	i	flight
H	hubs	j	route
R	Possible routes	t	day
R_h	Routes related to h is	h	hub
R'	Routes that have at least one empty flight		

- *Decreasing the number of aircraft needed, taking into account the possibility of changing the time of some flights*

One of the things that can be argued is the slight change in the time of some flights, so that some of the bottlenecks are removed and the flight sequence or cycles created will result in fewer aircraft. The mathematical model that should be used to solve this example is as follows:



- **Parameters and constant values:**

a_{ijt} : If route j covers the flight i on day t , it equals 1, otherwise 0
 N: Number of aircraft available

- **Variables:**

x_j : If route j is selected, it equals 1 and otherwise 0
 y_i : If flight i is done, it equals 1 and otherwise 0

- **Objective function:**

$$\text{Min } Z = \sum_{j \in R} x_j \quad (1)$$

- **Constraints:**

$$\sum_{i \in F_i} y_i = 1 \quad \forall i \in F \quad (2)$$

$$\sum_{j \in R_i} a_{ijt} x_j = y_i \quad \forall i \in (F \cup F'), \forall t \in T \quad (3)$$

$$\sum_{j \in R} x_j \leq N \quad (4)$$

$$x_j \text{ and } y_i = 0 \text{ or } 1 \quad (5)$$

Equation 1, which is the objective function, minimizes the total number of final routes, and since each route is allocated to an aircraft, the total number of aircraft needed will be minimized. Equation 2 ensures that each major flight is performed only once, and for flights based on which a new flight or flight with a change of time is made, only one of them is either the major or the new, not both. Equation 3 also ensures that each flight has only one route in the optimal solution per day, and in fact it ensures that the total number of routes selected will cover the optimal solution for all the flights needed. Equation 4 also ensures that the number of selected routes in the optimal answer, which is actually the number of aircraft needed, will not exceed the number of available planes. Equation 5 also indicates that the model variables are binary types, namely, zero and one.

- *Decreasing the number of aircraft needed, considering the possibility of the hub airport's displacement*

One of the things that can be argued is to change the hub airport to carry out maintenance services to eliminate some of the bottlenecks, and the flight sequences or cycles can lead to fewer aircraft.

The mathematical model that should be used to solve this example is as follows:

- **Parameters and constant values:**

a_{ijt} : If route j covers flight i on day t , it equals 1, otherwise 0
 k_{jh} : If route j corresponds to hub h it equals 1, otherwise 0
 N: Number of aircraft available

- **Variables:**

x_j : If route j is selected, it equals 1 and otherwise 0
 y_h : If hub h is selected, it equals 1 and otherwise 0

- **Objective function:**

$$\text{Min } Z = \sum_{j \in R} x_j \quad (6)$$



- **Constraints**

$$\begin{aligned}\sum_{h \in H} y_h &= 1 & (7) \\ \sum_{j \in R_h} k_{jh} a_{ijt} x_j &= y_h \quad \forall i \in F, \forall h \in H, \forall t \in T & (8) \\ \sum_{j \in R} x_j &\leq N & (9) \\ x_j \text{ and } y_h &= 0 \text{ or } 1 & (10)\end{aligned}$$

Equation 6, which is the objective function, minimizes the total number of final routes, and since each route is allocated to an aircraft, the total number of aircraft needed will be minimized. Equation 7 ensures that only one of the existing airports is selected as the hub. In fact, in the final solution, the best hub will be selected and introduced. Equation 8 also ensures that each flight has only one route in the optimal solution per day, and in fact ensures that the total number of routes selected in the optimal solution covers all flights needed per day. Of course, this control is performed on a series of cycles associated with a hub. Equation 9 also ensures that the number of selected routes in the optimal solution that is actually the number of aircraft needed will not exceed the number of available aircraft. Equation 10 also indicates that the model variables are binary types, namely, zero and one.

- *Mathematical model to reduce the number of aircraft needed, considering the possibility of removing a flight*

One of the things that can be argued is that it is possible to remove a flight in order for some of the flight bottlenecks removed, and the flight sequence or cycles can result in fewer aircraft. The mathematical model solves this problem as follows:

- **Parameters and constant values:**

a_{ijt} : If route j covers flight i on day t , it equals 1, otherwise 0

N : Number of aircraft available

K : Maximum number of removable flights

- **Variables:**

x_j : If route j is selected, it equals 1 and otherwise 0

y_i : If flight i is selected, it equal 1 and otherwise 0

- **Objective function:**

$$\text{Min } Z = \sum_{j \in R} x_j \quad (11)$$

- **Constraints:**

$$\sum_{i \in F_i} y_i = 1 \quad \forall i \in F \quad (12)$$

$$\sum_{j \in R_i} a_{ijt} x_j = y_i \quad \forall i \in (F \cup F'), \forall t \in T \quad (13)$$

$$\sum_{i \in F'} y_i \leq k \quad (14)$$

$$\sum_{j \in R} x_j \leq N \quad (15)$$

$$x_j \text{ and } y_i = 0 \text{ or } 1 \quad (16)$$

Equation 11, which is the objective function, minimizes the sum of the number of final routes, and since each route is allocated to an aircraft, the total number of aircraft needed will be



minimized. Equation 12 ensures that for every single flight, only its own or its virtual flight, which indicates the departure, is done. In fact, in the final answer there is either the major flight or the virtual flight, not both. Equation 13 ensures that each flight only has one of the routes in the optimal solution per day, and in fact ensures that the total number of routes selected in the optimal solution covers all flights needed per day. Equation 14 also ensures that the number of flights removed from does not exceed a specified number k . The specified number at the beginning of the model is given as a constant value to the model so that the model does not eliminate the major flights by its own. Of course, it should be noted that, for example the model might not give a solution for $k = 1$, that is one cannot be reduce the number of routes needed, and thus reduce the number of aircraft needed by removing only one flight. So, with $k = 2$ or more, one can solve the model so that the solution is obtained. Equation 15 also ensures that the number of routes selected in the optimal solution, which is actually the number of aircraft needed, does not exceed the number of available aircraft. Equation 16 also indicates that the model variables are binary types, namely, zero and one.

- **Reducing the number of aircraft needed with the possibility of adding non-passenger flights**

One of the things that can be argued is adding a non-passenger flight on some routes to remove some of the flight bottlenecks, and the flight sequences or cycles created will result in fewer aircraft. One of the conditions for creating a two-day cycle is that the last destination of the first day is the same as the first origin of the second day. This means that a plane landed at the airport at the end of the first day and should start its flights from the airport tomorrow morning.

The mathematical model for solving this problem is as follows:

- **Parameters and constant values:**

e_j : If route j has at least one empty flight, it equals 1, otherwise 0

a_{ijt} : If route j covers flight i on day t , it equals 1, otherwise 0

N : Number of aircraft available

- **Variables:**

x_j : If route j is selected, it equals 1, otherwise 0

- **Objective function:**

$$\text{Min } Z = \sum_{j \in R'} e_j x_j + \sum_{j \in R} x_j \quad (17)$$

- **Constraints:**

$$\sum_{j \in R_i} a_{ijt} x_j = 1 \quad \forall i \in F, \forall t \in T \quad (18)$$

$$\sum_{j \in R'} e_j x_j \geq 1 \quad (19)$$

$$\sum_{j \in R} x_j \leq N \quad (20)$$

$$x_j = 0 \text{ or } 1 \quad (21)$$

Equation 17, which is the objective function, consists of two parts. The first part computes the total number of routes in which there is at least one empty flight, and the second part is the total number of selected routes in the final solution. In fact, we also want to minimize the total number of endpoints and also want to use the least empty flights. Equation 18 ensures that each flight is covered by only one of the routes. Equation 19 ensures that there is at least one empty flight in the final solution. If this equation is not in the model, according to the first part of the objective



function, the mathematical model tries to reduce the number of empty flying routes to zero, and if it equals zero, the problem is cleared, meaning that there is no empty flights, while we want to check the possibility of saving aircraft by adding at least one empty flight. Equation 20 ensures that the number of routes selected and the number of aircraft needed does not exceed the number of available aircraft. Equation 21 also defines the type of variables as binary.

Solving a numerical example to reduce the number of aircraft needed, considering the possibility of adding non-passenger flights

- ***Mathematical model***

As discussed in Section 5.4, one of the things that can be argued is adding a non-passenger flight on some routes to remove some of the flight bottlenecks, and the flight sequence or cycles created will result in fewer aircraft. If the last destination of the first day is not the same as the first origin of the second day, what effect will it have on the flight network? In other words, what will happen if this aircraft starts the second day flights from another airport?

Consider the flight information of an airline company:

Table 2. Flight Information of an Airline

Flight No.	Origin	Departure	Destination	Landing time	Duration
۱۲۵	TEH	۰۷:۲۵	ESF	۱۲:۵۵	۰۵:۳۰
۱۱۰	SHI	۰۸:۱۰	TEH	۱۰:۴۰	۰۲:۳۰
۱۱۳	MSH	۰۹:۱۰	TEH	۱۲:۱۰	۰۳:۰۰
۱۳۱	TEH	۰۹:۳۰	SHI	۱۲:۰۰	۰۲:۳۰
۱۰۵	ESF	۱۲:۵۰	TEH	۱۸:۲۰	۰۵:۳۰
۱۳۸	TEH	۱۲:۳۰	SAR	۱۴:۰۰	۰۱:۳۰
۱۱۱	SHI	۱۳:۱۰	TEH	۱۵:۴۰	۰۲:۳۰
۱۱۴	MSH	۱۴:۳۰	TEH	۱۷:۳۰	۰۳:۰۰
۱۱۸	SAR	۱۵:۱۰	TEH	۱۶:۴۰	۰۱:۳۰
۱۳۵	TEH	۱۵:۱۰	MSH	۱۸:۱۰	۰۳:۰۰
۱۳۳	TEH	۱۸:۰۵	SHI	۲۰:۳۵	۰۲:۳۰
۱۳۶	TEH	۱۸:۱۰	MSH	۲۱:۱۰	۰۳:۰۰

Suppose the return time is 45 minutes (that is, the minimum interval between two consecutive flights). It is very difficult to generate a sequence of consecutive routes that can be assigned to an aircraft throughout the day. The number of possible routes is also very high. On the other hand, one of the above sequences should be finally selected, because for example, flight 131 should only be done once, not twice. Also, the total flight time of each sequence should be within the limits of the permissible daily flight of an aircraft in accordance with existing standards. For example, an aircraft can fly up to 8 hours a day. Such limitations make it difficult to generate and select possible routes. Now, to build the three-day routes, we take the condition that the last destination of the first is the same as the first origin of the next day. In this case, the number of three-day routes is 29791 and the number of correct routes to enter the mathematical model is 7245.

Gams software was used. As a result, after solving the model, 7 final routes (3-day cycle) were obtained as the optimal solution. The number of selected and optimal cycles is:

$$۵۱۵-۵۵۶۴-۸۱۲۸-۱۳۴۷۱-۱۵۴۰۸-۱۹۵۲۴-۲۳۶۹۶$$



Each of these cycles is allocated to an aircraft, and the number of first, second and third day flight of each aircraft is shown in Table 3. Therefore, in order to run this flight network, we will need at least 7 aircraft in 3 consecutive days (3-day cycle), and lower than this is not feasible. The number of aircraft needed shows that one aircraft is saved by adding the possibility of performing non-passenger flights (in the absence of a non-passenger flight, the number of routes will be 8). Of course, of these 7 routes, there are 2 routes with empty flight shown in Table 2.

Routes Nos. 13471 and 15408 include non-passenger flights. On route No. 13471, the last flight of the first day with number 133 will be from Tehran to Shiraz, while in the second day flight No. 125 should be done, which is from Tehran to Isfahan. Therefore, the last destination of the first day is not the same as the first origin of the second day (Shiraz and Tehran). As a result, a non-passenger flight from Shiraz to Tehran should be carried out so that the aircraft can do flight 125 from Tehran on the second day. This flight must be made without passengers or at the end of the first day or the beginning of the second day.

At the end of the first day route 15408 also has the same problem with the beginning of the second day and a non-passenger flight from Tehran to Shiraz should be carried out.

Apart from the two above-mentioned routes, the rest of the routes have the correct cycle. The end of the first day is the same as the beginning of the second day. Also, the end of the second day is the same as the beginning of the third day. So, in the final optimal solution, of the 7 selected routes, there are 5 routes for passenger flights and 2 non-passenger flights.

Table 3. The final solution to the mathematical model with the possibility of performing non-passenger flights

3-day Flight No.	First day route No.	First day flight No.	Second day route No.	Second day flight No.	Third day route No.	Third day flight No.
010	1	120	17	100	19	138-118
0074	6	110-130	20	114	20	131-111-133
8128	9	113	10	131-111-133	6	110-130
13471	10	131-111-133	1	120	17	100
10408	17	100	2	110	1	120
19024	21	138-118-136	10	113-130	20	114
23796	20	114	21	138-118-136	12	113-136

A detailed analysis should be made whether or not 2 non-passenger flights are acceptable in the flight network. It should be checked that if reducing an aircraft by performing two non-passenger flights is cost-effective for the flight network? If there are restrictions on access to the airline fleet, we may accept flights scheduled with a lesser airplane with 2 non-passenger flights.

- ***Genetic algorithm for solving the mathematical model***

Consider the information in Table 2. There are 12 defined flights that ultimately have 7245 3-day flight cycles to be completed with the minimum number of aircraft. After several times the algorithm was executed, we reached the same number of 7 replications, which was the solution to the mathematical model. The maximum number of replications for the algorithm was defined as 100,000, and the initial population size was 30. The percentage of mutation and the intersection of a point were also 0.5.

Each time the algorithm was executed due to the randomness of the production of the primary population, random selection of the selected gene for the mutation operator, as well as random selection of the selected gene for the intersection operator; the number of replications needed to reach the acceptable solution was different. In some cases, after 4292 replications, we arrived at the solution and in some cases after 70,205 replications. The result of the 8-time execution of the algorithm is shown in Table 4.

Table 4: Results of the Genetic Algorithm for Information of Table 1 (Possibility to carry out non-passenger flights)

No. of replications	No. of replication to reach the solution	Runtime for reach eh solution (second)	No. 7 of the selected cycle (No. Of chromosome genes)	Objective function value (Competency function)	No. of maintenance and repair opportunities in the hub airport
۱	۳۰۰۴۲	۱۰۸۲	-۱۷۴۷۸-۲۳۰۹۳ -۱۰۱۷۶-۱۰۶۲۰ ۰۹۳-۰۳۱۲-۱۰۰۹۱	-11	11
۲	۷۰۲۰۰	۲۰۲۷	-۲۳۰۷۰-۲۹۱۰۹ -۱۳۹۳۶-۱۰۸۷۳ ۱۸۰-۳۴۰۲-۹۴۰۹	-11	11
۳	۱۶۷۰۸	۶۰۳	-۲۳۰۷۶-۲۷۸۲۴ -۱۴۹۰۰-۱۰۴۷۰ ۲۸۳-۳۳۹۴-۱۰۳۸۳	-11	11
۴	۱۱۱۷۸	۴۰۲	-۲۳۱۹۸-۲۸۶۳۸ -۱۴۹۱۰-۱۰۶۲۰ ۱۷-۳۷۶۶-۹۱۴۰	-11	11
۵	۶۱۰۴	۲۲۰	-۱۷۶۰۲-۲۳۹۷۱ -۱۴۰۲۰-۱۰۸۴۲ ۷۶۳-۰۳۱۶-۹۶۲۰	-11	11
۶	۴۲۹۲	۱۰۰	-۱۷۰۷۱-۲۳۸۲۰ -۱۳۹۳۶-۱۰۸۷۳ ۴-۶۸۹۷-۹۲۶۷	-11	11
۷	۴۰۹۰	۱۶۰	-۲۳۸۳۷-۲۹۰۰۲ -۱۳۷۶۸-۱۰۷۸۰ ۰۲۱-۲۸۹۸-۹۲۸۱	-11	11
۸	۴۷۸۹	۱۷۲	-۱۸۰۱۱-۲۳۰۸۹ -۱۲۰۱۰-۱۶۱۲۱ ۱۸۰-۰۲۸۰-۱۱۱۷۲	-11	11


As can be seen in Table 4, there are several solutions to this problem. As a result, it can be said that the main problem is the optimal multiple solution. The value of the competency function is also -11 in all replications, which actually indicates that there are 11 repair and maintenance opportunities in all the solutions obtained. Of course, in some cases, after the completion of 100,000 times the algorithm was repeated, no acceptable solution was reached. Also, the average runtime for each algorithm in this case was 0.036 seconds (P4, Ram: 4G, Cpu: 2.4G-Dual).



RESULTS

Optimization of scheduling flights can be accomplished for several purposes: minimizing the number of aircraft required, minimizing flight costs, maximizing the total flight hours, maximizing the number of maintenance opportunities at the hub, and so on. For this purpose, a model is proposed in which the current flight plan is considered as the input of the model and then, by solving the mathematical model, the best suggestion is presented for minor variations in the flight network. These minor changes can include changes in the time of some flights. Of course, this mathematical model offers only one solution, and it is necessary to make a cost-benefit analysis for the airline company. On the other hand, the proposed mathematical model is NP-Hard and cannot be solved by increasing the dimensions of the problem with existing software and computers. For this purpose, a genetic algorithm was used and a program was written in MATLAB. Numerical examples as inputs of mathematical models were solved using the genetic algorithm and the obtained solutions were able to confirm the solutions of mathematical models. The proposed genetic algorithm has reached a satisfactory solution in a short and acceptable time suggesting that this algorithm can be used in similar problems but with larger dimensions.

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