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OBTAINING EFFICIENT SOLUTIONS FOR FUZZY MULTI-OBJECTIVE TRANSPORTATION PROBLEMS BY IMPLEMENTING A-CUT SET

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ABSTRACT

The goal of linear programming problems is to minimize costs or maximize profits, but in general, problems that are formulated in reality are multi-objective, and these goals are often measured at different scales and are incompatible with each other. In practice, the ideal solution to a multi-objective problem is impossible in most cases. When the goals of the problem are in conflict, such solutions cannot be achieved. For this purpose, instead of the ideal solution, the concept of the correct solution is introduced. In this paper, we introduce a method for solving fuzzy multi-objective transportation problems where the cost coefficients of the objective functions, suppliers and demands are expressed as fuzzy numbers. The fuzzy multi-objective transportation problem is transformed into multi-objective interval transportation problem by using α-cut set of a fuzzy number. The multi-objective interval transportation problem is converted into several single objective interval transportation problems and are solved by separation method. Then efficient solutions are obtained by interactive procedure. A numerical example is presented to illustrate the efficiency of the method.

Keywords: *Multi-Objective Transportation Problems, Fuzzy Multi-Objective Transportation Problems, Multi-Objective Interval Transportation Problems, Separation Method, Efficient Solution, A-Cut Set, Zero-Point Method.*

INTRODUCTION

Transportation problems are used in economic and social activities and is important to operational research and management sciences. Transportation problems are a form of linear programming problems with constraints specific to the structure. The objective function of classic transportation is to minimize costs, but in general, problems that are formulated in reality are multi-objective problems and these objectives are often measured at different scales and are incompatible with each other. In practice, the ideal solution to multi-objective problems is impossible in most cases. When the objectives of the multi-objective transport problem are in conflict, such solutions cannot be achieved. For this purpose, instead of the ideal solution, a numerical solution is presented. A number of techniques have been developed to find a compromise solution to multi-objective optimization problems. The reader is referred to the recent books by Miettinen (Miettinen, 1999) about the theory and algorithms for multi-objective optimization problems and Multi-objective decision making: Theory and methodology by Chankong and Haimes (Chankong & Haimes, 1983) and multiple criteria decision making by Zeleny (Zeleny, 1982). Das, et.al., proposed a method for solving multi-objective transportation problem with interval cost, source and destination parameters (Das et al., 1999). Hussien, proposed a method for complete solutions of multiple objective transportation problem with possibilistic coefficients (Hussien, 1998). Often, in multi-objective transportation problem, the

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coefficients of the objective functions and supply and demand are fuzzy data that must be determined by the decision maker. The decision maker then obtains the solution to the problem by analyzing the data with the necessary methods. Valued books by Zimmermann (Zimmermann, 1996) about Fuzzy set theory and its applications and Zadeh and Bellman (Bellman & Zadeh, 1970) about decision making in a fuzzy environment and fuzzy sets and interactive multi-objective optimization by Sakawa (Sakawa, 1993) are recommed. In 2005, an algorithm was proposed by Omar and Yunes for solving multi–objective transportation problems using fuzzy factors (Ammar & Youness, 2005). Sheikhi in 2014 introduced a novel algorithm for solving two-objective fuzzy transportation problems (Sheikhi, 2014). Abd Wahed has solved multi-objective transportation problems under fuzziness (Abd Wahed, 2001). Kikuchi, introduced “A method of defuzzify the number: transportation problem application” (Kikuchi, 2000). In 2010, a new method using ranking of generalized fuzzy numbers was presented by Amit Kumar, et.al., (2010). In 2011, a new method was presented by Pendian for solving two objective transportation problems (Pandian & Anuradha, 2011). Amit and Pushpinder (2010) used ranking of generalized trapezoidal fuzzy numbers (Kumar et al., 2010).

MULTI-OBJECTIVE TRANSPORTATION PROBLEM

The multi-objective transportation problem (MOTP), when the objective functions coefficients are classic number, and the constraints are deterministic, i.e., the parameters a_i and b_j are deterministic, is as follows:

(MOTP) Minimize $Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \quad for\ k = 1,2,\dots K$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i \quad for\ \ i = 1,2,..m$ (2.1)

$\sum_{i=1}^{m} x_{ij} = b_j \quad for\ \ j = 1,2,\dots,n$ (2.2)

$x_{ij} \geq 0,\ \ i = 1,2,\dots,m;\ j = 1,2,\dots,n$ (2.3)

We assume that $a_i > 0,\ b_j > 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n$; and total demand equals to total supply, i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

Definition 2.1:(Chankong & Haimes, 1983) A set $S = \{x_{ij}^{0}, i = 1,2,\dots,m; j = 1,2,\dots,n\}$ is said to be feasible to the problem (MOTP) if S satisfies the conditions (2.1) to (2.3).

Definition 2.2: (Chankong & Haimes, 1983) A point $\bar{x} \in S$ is said to be an efficient point if and only if there exists no $x \in S$ such that for some $s \in \{1,\dots,k\}$:

$$Z^s(x) < Z^s(\bar{x}) \quad and\ \ Z^i(x) \leq Z^i(\bar{x}) \quad for\ all\ i = 1,2,\dots,k\ , i \neq s \qquad (2.4)$$

Definition 2.3: (Zimmermann, 1996) Let R be real numbers set, $\tilde{a}$ fuzzy number is a map with following conditions:

1) $\mu_{\tilde{a}}$ is continuous.
2) $\mu_{\tilde{a}}$ on $[a_1, a_2]$ is increasing and continuous.
3) $\mu_{\tilde{a}}$ on $[a_3, a_4]$ is decreasing and continuous.

Where a_1, a_2, a_3 and a_4 are real numbers and fuzzy number shown as $\tilde{a} = [a_1, a_2, a_3, a_4]$ is called trapezoidal fuzzy number.

Definition 2.4: (Sheikhi, 2014) A trapezoidal fuzzy number $\tilde{a} = [a_1, a_2, a_3, a_4]$ can be represented as an interval number form as follows:

$$[a_1, a_2, a_3, a_4] = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha];\ 0 \le \alpha \le 1. \tag{2.5}$$

Definition 2.5: (Zimmermann, 1996) if $\tilde{a}$ is a trapezoidal fuzzy number, then the membership function for the fuzzy number $\tilde{a}$ is as follows

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & , \quad x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , \quad a_1 \le x \le a_2 \\ 1 & , \quad a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & , \quad a_3 \le x \le a_4 \\ 0 & , \quad x \ge a_4 \end{cases} \tag{2.6}$$

Definition 2.6: (Bellman & Zadeh, 1970) α-cut set of a fuzzy number ã is shown by A_α and is defined as follows:

$$A_\alpha = \{x | \mu_{\tilde{A}}(x) \ge \alpha\} = [a_\alpha^l, a_\alpha^u] \tag{2.7}$$

Definition 2.7: (Amit et al., 2010) Robust ranking technique which satisfies compensation, linearity, and additivity properties and provides results consisting human intuition. If $\tilde{a}$ is a fuzzy number then the robust ranking is defined by

$$R(\tilde{a}) = \frac{1}{2}\int_0^1 (a_\alpha^l + a_\alpha^u) d\alpha, \tag{2.8}$$

where $\mathrm{a}_\alpha^\mathrm{l}$ and $\mathrm{a}_\alpha^\mathrm{u}$ are the lower bound and the upper bound of the α −cut set of the fuzzy number $\tilde{a}$, respectively.

Definition 2.8: (Dubois & Prade, 1978) Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{a} \ominus \tilde{b} = (a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$

(iii) $k\tilde{a} = k(a_1, a_2, a_3, a_4) = \begin{cases} (ka_1, ka_2, ka_3, ka_4) & for\ k \ge 0 \\ (ka_4, ka_3, ka_2, ka_1) & for\ k < 0 \end{cases}$

(iv) $\tilde{a} \otimes \tilde{b} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (t_1, t_2, t_3, t_4)$

where $t_1 = \text{minimum}\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$;

$t_2 = \text{minimum}\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$;

$t_3 = \text{maximum}\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$;

$t_4 = \text{maximum}\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$.

FUZZY MULTI-OBJECTIVE TRANSPORTATION PROBLEM

Consider the following fuzzy multi-objective transportation problem (FMITP) where the objective function coefficients and the constraints are fuzzy numbers, the fuzzy multi-objective transportation problem is defined as follows:

(FMOTP) Minimize $\tilde{Z}^k \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{k}\tilde{x}_{ij} \quad for\ k = 1,2,\dots K$

$$\text{Subject to } \sum_{j=1}^{n}\tilde{x}_{ij} = \tilde{a}_i \quad for\ i = 1,2,..m \tag{3.1}$$

$$\sum_{i=1}^{m}\tilde{x}_{ij} = \tilde{b}_j \quad for\ j = 1,2,\dots,n \tag{3.2}$$

$$\tilde{x}_{ij} \geq \tilde{0},\ \ i = 1,2,\dots,m; j = 1,2,\dots,n \tag{3.3}$$

We assume that $a_i > 0,\ b_j > 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n$; and total demand equals to total supply, i.e. $\sum_{i=1}^{m}\tilde{a}_i = \sum_{j=1}^{n}\tilde{b}_j$.

Definition 3.1: A set $\tilde{S} = \{\tilde{x}_{ij}^{0}, i = 1,2,\dots,m; j = 1,2,\dots,n\}$ is said to be feasible to the problem (MOTP) if $\tilde{S}$ satisfies the conditions (3.1) to (3.3).

Definition 3.2: A trapezoidal fuzzy number $[\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4] \in \tilde{S}$ is said to be an efficient fuzzy point if and only if there exists no $[a_1, a_2, a_3, a_4] \in \tilde{S}$ such that for some $s \in \{1,\dots,k\}$:

$$Z^s(a_t) < Z^s(\bar{a}_t) \ \text{ and } \ Z^i(a_t) \leq Z^i(\bar{a}_t) \ \text{ for all } i = 1,2,\dots,k, i \neq s \text{ and } t = 1,2,3,4. \tag{3.4}$$

INTERVAL TRANSPORTATION PROBLEM

When the objective function coefficients, the source parameters and destination parameters are in the form of interval, the interval multi-objective transportation problem (IMTP) is then as follows:

$$\text{(ITP)} \quad \text{Minimize} \quad [Z_1, Z_2] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$$

$$\text{Subject to } \sum_{j=1}^{n}[x_{ij}, y_{ij}] = [a_i, p_i] \quad for\ i = 1,2,..m \tag{4.1}$$

$$\sum_{i=1}^{m}[x_{ij}, y_{ij}] = [b_j, q_j] \quad for\ j = 1,2,\dots,n \tag{4.2}$$

$$x_{ij} \geq 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n \ and\ are\ integers. \tag{4.3}$$

Where c_{ij}^{k} and d_{ij}^{k} are positive real numbers for all i , j and k , a_i and p_i are positive real numbers for all i , b_j and q_j are positive real numbers for all j. We assume that total demand equals to total supply, i.e. $\sum_{i=1}^{m}a_i = \sum_{j=1}^{n}b_j\ and\ \sum_{i=1}^{m}p_i = \sum_{j=1}^{n}q_j$.

Now, we use the following Theorem which finds a relation between optimal solutions of a interval transportation problem and a pair of induced transportation problems, and is also used in the proposed method.

Theorem 4.1: (Pandian & Natarajan, 2010) If the set { y_{ij}^0 for all i and j } is an optimal solution of the upper bound transportation problem (UBITP) of interval transportation problem (ITP) where

$$\text{(UBITP) minimize } Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}$$

Subject to $\sum_{j=1}^{n} y_{ij} = p_i \quad for\ i = 1,2,\dots,m$ (4.4)

$\sum_{i=1}^{n} y_{ij} = q_j \quad for\ j = 1,2,\dots,n$ (4.5)

$y_{ij} \geq 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n,$ (4.6)

and the set $\{\, x_{ij}^0\ for\ all\ i\ and\ j \,\}$ is an optimal solution of the lower bound transportation problem (LBITP) of (ITP) where

$$\text{(LBITP) minimize } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i \quad for\ i = 1,2,\dots,m$ (4.7)

$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ j = 1,2,\dots,m$ (4.8)

$x_{ij} \geq 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n.$ (4.9)

Then the set of intervals { $[x_{ij}^0\ , y_{ij}^0\ ]\ for\ all\ i\ and\ j$ } is an optimal solution of the problem (ITP) provided $x_{ij}^0\ \leq\ y_{ij}^0$, for all i and j.

MULTI-OBJECTIVE INTERVAL TRANSPORTATION PROBLEM

When the objective function coefficients, the source parameters and destination parameters are in the form of interval, the interval multi-objective transportation problem (IMTP) is defined as follows:

$$\text{(IMTP)} \quad \text{Minimize} \quad [Z_1^k, Z_2^k] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}^k, d_{ij}^k] \otimes [x_{ij}, y_{ij}]\ for\ k = 1,2,\dots,K$$

Subject to $\sum_{j=1}^{n}[x_{ij}, y_{ij}] = [a_i, p_i] \ \ for\ \ i = 1,2,..m$ (5.1)

$\sum_{i=1}^{m}[x_{ij}, y_{ij}] = [b_j, q_j] \ \ for\ \ j = 1,2,\dots,n$ (5.2)

$x_{ij} \geq 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n\ and\ are\ integers.$ (5.3)

Where c_{ij}^k and d_{ij}^k are positive real numbers for all i , j and k , a_i and p_i are positive real numbers for all i and b_j and q_j are positive real numbers for all j. We assume that total demand equals to total supply, i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\ \ and\ \ \sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j$.

Definition 4.1:(Pandian & Natarajan, 2010) A feasible solution $\{[x_{ij}, y_{ij}], for\ i = 1,2,\dots,m\ \ and\ j\ =\ 1,2,\dots,n\}$ of the problem (IMTP) is said to be an optimal solution of (IMTP) if

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}^{k}, d_{ij}^{k}] \otimes [x_{ij}, y_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}^{k}, d_{ij}^{k}] \otimes [u_{ij}, v_{ij}] \quad (5.4)$$

for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$ and for all feasible $\{ [u_{ij}, v_{ij}]\ for\ i = 1,2,\ldots,n$ and $j = 1,2,\ldots,n \}$.

PROPOSED METHOD

We now propose a new method for solving multi-objective transportation problem with fuzzy numbers. The steps of the proposed method are as follows:

Step 1: Using α-cut set of a fuzzy number, we then convert the given fuzzy transportation problem into an interval multi-objective transportation problem (IMTP).

Step 2: Choose the first objective of the transportation problem and solve this problem with single-objective transportation problem with conditions of Step 1 by using separation method (Pandian & Natarajan, 2010).

Step 3: Same as previous step perform K times for K different objective functions. If all the solutions are the same, then one of them is ideal solution, and in this case the algorithm ends. Otherwise, go to step 4

Step 4: Obtain the fuzzy efficient solutions corresponding to optimal solutions for K single-objective interval transportation problem.

Step 5: Construct a linear compromise function of the problem as follows:

$$\tilde{Z}^{L} = \sum_{k=1}^{K}\frac{1}{R(Z^{k^*})}\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}_{ij}^{k}\,\tilde{x}_{ij}$$

Then determine its optimal solution and its corresponding fuzzy efficient solution.

Step 6: Obtain fuzzy efficient solutions corresponding to adjacent extreme points to optimal solution in step5.

Step 7: If decision maker selects the preferred solution to fuzzy multi-objective transportation problem using the fuzzy efficient solutions of the previous steps, the algorithm ends. Otherwise, go to step 8.

Step 8: Find the adjacent extreme points with the fuzzy efficient solutions and add fuzzy efficient solutions corresponding to the adjacent extreme points to fuzzy efficient solutions. Then go to step 7.

NUMERICAL EXAMPLE

Let us consider a fuzzy multi-objective transportation problem with the following characteristics

Minimize $\tilde{Z}^{k} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{k}\tilde{x}_{ij} \quad for\ k = 1,2$

Subject to $\sum_{j=1}^{n}\tilde{x}_{ij} = \tilde{a}_{i} \quad for\ \ i = 1,2,..m$

$\sum_{i=1}^{m}\tilde{x}_{ij} = \tilde{b}_{j} \quad for\ \ j = 1,2,\ldots,n$

$\tilde{x}_{ij} \geq \tilde{0}, \ \ i = 1,2,\ldots,m; j = 1,2,\ldots,n$

where $\tilde{c}_{11}^{1} = (4,6,10,11)$, $\tilde{c}_{12}^{1} = (36,38,41,44)$, $\tilde{c}_{13}^{1} = (12,13,15,18)$, $\tilde{c}_{14}^{1} = (8,10,11,12)$,

$\tilde{c}_{21}^1 = (6,7,9,10)$, $\tilde{c}_{22}^1 = (8,10,14,16)$, $\tilde{c}_{23}^1 = (48,50,53,56)$, $\tilde{c}_{24}^1 = (2,3,4,5)$,

$\tilde{c}_{31}^1 = (52,55,58,60)$ $\tilde{c}_{32}^1 = (0,2,3,6)$, $\tilde{c}_{33}^1 = (14,16,18,20)$, $\tilde{c}_{34}^1 = (16,18,21,24)$,

$\tilde{c}_{11}^2 = (20,22,24,26)$, $\tilde{c}_{12}^2 = (4,6,7,10)$, $\tilde{c}_{13}^2 = (0,1,2,4)$, $\tilde{c}_{14}^2 = (9,12,13,15)$,

$\tilde{c}_{21}^2 = (10,11,14,16)$, $\tilde{c}_{22}^2 = (32,34,36,38)$, $\tilde{c}_{23}^2 = (1,4,5,6)$, $\tilde{c}_{24}^2 = (0,1,1,2)$,

$\tilde{c}_{31}^2 = (2,4,5,7)$, $\tilde{c}_{32}^2 = (44,46,50,52)$, $\tilde{c}_{33}^2 = (8,10,12,15)$, $\tilde{c}_{34}^2 = (7,10,11,13)$,

2-Supplies: $\tilde{a}_{\mathbf{1}} = (30,34,38,44)$, $\tilde{a}_2 = (46,50,54,56)$, $\tilde{a}_3 = (34,36,41,46)$

3-Demand: $\tilde{b}_{\mathbf{1}} = (40,43,45,50)$, $\tilde{b}_2 = (16,20,22,25)$, $\tilde{b}_3 = (28,30,34,37)$,

$\tilde{b}_4 = (26,27,32,34)$

Convert the given fuzzy transportation problem into an interval multi-objective transportation problem (IMTP)

$\tilde{c}_{11}^1 = [4+2\alpha, 11-\alpha]$, $\tilde{c}_{12}^1 = [36+2\alpha, 42-\alpha]$, $\tilde{c}_{13}^1 = [12+\alpha, 18-3\alpha]$, $\tilde{c}_{14}^1 = [8+2\alpha, 12-\alpha]$,

$\tilde{c}_{21}^1 = [6+\alpha, 10-\alpha$, $\tilde{c}_{22}^1 = [8+2\alpha, 16-2\alpha]$, $\tilde{c}_{23}^1 = [48+2\alpha, 56-3\alpha]$, $\tilde{c}_{24}^1 = [2+\alpha, 5-\alpha]$,

$\tilde{c}_{31}^1 = [52+3\alpha, 60-2\alpha]$, $\tilde{c}_{32}^1 = [2\alpha, 6-3\alpha]$, $\tilde{c}_{33}^1 = [14+2\alpha, 20-2\alpha]$,

$\tilde{c}_{34}^1 = [16+2\alpha, 24-3\alpha]$,

$\tilde{c}_{11}^2 = [20+2\alpha, 26-2\alpha]$, $\tilde{c}_{12}^2 = [4+2\alpha, 10-3\alpha]$, $\tilde{c}_{13}^2 = [\alpha, 4-2\alpha]$, $\tilde{c}_{14}^2 = [9+3\alpha, 15-2\alpha]$,

$\tilde{c}_{21}^2 = [10+\alpha, 16-2\alpha]$, $\tilde{c}_{22}^2 = [32+2\alpha, 38-2\alpha]$, $\tilde{c}_{23}^2 = [1+3\alpha, 6-\alpha]$, $\tilde{c}_{24}^2 = [\alpha, 2-\alpha]$,

$\tilde{c}_{31}^2 = [2+2\alpha, 7-2\alpha]$, $\tilde{c}_{32}^2 = [44+2\alpha, 52-2\alpha]$, $\tilde{c}_{33}^2 = [8+2\alpha, 15-3\alpha]$,

$\tilde{c}_{34}^2 = [7+3\alpha, 13-2\alpha]$

2-Supplies: $\tilde{a}_1 = [30+4\alpha, 44-6\alpha]$, $\tilde{a}_2 = [46+4\alpha, 56-2\alpha]$, $\tilde{a}_3 = [34+2\alpha, 46-5\alpha]$

3-Demand: $\tilde{b}_1 = [40+3\alpha, 50-5\alpha]$, $\tilde{b}_2 = [16+4\alpha, 25-3\alpha]$, $\tilde{b}_3 = [28+2\alpha, 37-3\alpha]$,

$\tilde{b}_4 = [26+\alpha, 34-2\alpha]$.

The single objective interval transportation problem with the first objective function is given below:

$\tilde{c}_{11}^1 = [4+2\alpha, 11-\alpha]$, $\tilde{c}_{12}^1 = [36+2\alpha, 42-\alpha]$, $\tilde{c}_{13}^1 = [12+\alpha, 18-3\alpha]$, $\tilde{c}_{14}^1 = [8+2\alpha, 12-\alpha]$,

$\tilde{c}_{21}^1 = [6+\alpha, 10-\alpha]$, $\tilde{c}_{22}^1 = [8+2\alpha, 16-2\alpha]$, $\tilde{c}_{23}^1 = [48+2\alpha, 56-3\alpha]$, $\tilde{c}_{24}^1 = [2+\alpha, 5-\alpha]$,

$\tilde{c}_{31}^1 = [52+3\alpha, 60-2\alpha]$, $\tilde{c}_{32}^1 = [2\alpha, 6-3\alpha]$, $\tilde{c}_{33}^1 = [14+2\alpha, 20-2\alpha]$, $\tilde{c}_{34}^1 = [16+2\alpha, 24-3\alpha]$

2-Supplies: $\tilde{a}_1 = [30+4\alpha, 44-6\alpha]$, $\tilde{a}_2 = [46+4\alpha, 56-2\alpha]$, $\tilde{a}_3 = [34+2\alpha, 46-5\alpha]$

3-Demand: $\tilde{b}_1 = [40+3\alpha, 50-5\alpha]$, $\tilde{b}_2 = [16+4\alpha, 25-3\alpha]$, $\tilde{b}_3 = [28+2\alpha, 37-3\alpha]$,

$$\tilde{b}_4 = [26 + \alpha, 34 - 2\alpha]$$

The UBITP of the first objective interval transportation problem is defined as below

Destination→ source↓	1	2	3	4	supply
1	$11 - \alpha$	$42 - \alpha$	$18 - 3\alpha$	$12 - \alpha$	$44 - 6\alpha$
2	$10 - \alpha$	$16 - 2\alpha$	$56 - 3\alpha$	$5 - \alpha$	$56 - 2\alpha$
3	$60 - 2\alpha$	$6 - 3\alpha$	$20 - 2\alpha$	$24 - 3\alpha$	$46 - 5\alpha$
demand	$50 - 5\alpha$	$25 - 3\alpha$	$37 - 3\alpha$	$34 - 2\alpha$	

Now, using the zero-point method (Gaurav et al., 2015), an optimal solution to the UBITP is found to be:

$$x_{11} = 28 - 5\alpha, x_{13} = 16 - \alpha\,, x_{21} = 22, x_{24} = 34 - 2\alpha\,, x_{32} = 25 - 3\alpha, x_{33} = 21 - 2\alpha.$$

The LBITP of the first objective interval transportation problem is given below

Destination→ source↓	1	2	3	4	supply
1	$4 + 2\alpha$	$36 + 2\alpha$	$12 + \alpha$	$8 + 2\alpha$	$30 + 4\alpha$
2	$6 + \alpha$	$8 + 2\alpha$	$48 + 2\alpha$	$2 + \alpha$	$46 + 4\alpha$
3	$52 + 3\alpha$	2α	$14 + 2\alpha$	$16 + 2\alpha$	$34 + 2\alpha$
demand	$40 + 3\alpha$	$16 + 4\alpha$	$28 + 2\alpha$	$26 + \alpha$	

Now, using the zero-point method, an optimal solution to the LBITP is found to be:

$$x_{11} = 20, x_{13} = 10 + 4\alpha\,, x_{21} = 20 + 3\alpha\,, x_{24} = 26 + \alpha\,, x_{32} = 16 + 4\alpha\,, x_{33} = 18 - 2\alpha$$

Therefore, the optimal solution to the first objective interval transportation problem is obtained as:

$$x_{11} = [20,28 - 5\alpha];\ x_{13} = [10 + 4\alpha, 16 - \alpha]\,;\ x_{21} = [20 + 3\alpha, 22]$$

$$x_{24} = [26 + \alpha, 34 - 2\alpha]\,; x_{32} = [16 + 4\alpha, 25 - 3\alpha];\ x_{33} = [18 - 2\alpha, 21 - 2\alpha].$$

Thus, the fuzzy optimal solution for the first objective interval transportation problem is

$$\tilde{x}_{11} = (20,20,23,28);\ \tilde{x}_{13} = (10,14,15,16)\,;\ \tilde{x}_{21} = (20,22,22,23)$$

$$\tilde{x}_{24} = (26,27,32,34)\,; \tilde{x}_{32} = (16,20,22,25);\ \tilde{x}_{33} = (16,18,19,21)$$

The fuzzy objective value is $\tilde{Z}^1 = (596,865,1189,1566)$ and $\mathrm{R}(\tilde{Z}^1) = 1054$.

Similarly, the optimal solution to the second objective interval transportation problem is defined as:

$$x_{11} = [16 + 4\alpha, 25 - 3\alpha];\ x_{13} = [14,19 - 3\alpha]\,;\ x_{21} = [4,6 + \alpha]$$

$$x_{24} = [14 + 2\alpha, 18]\,; x_{32} = [26 + \alpha, 34 - 2\alpha];\ x_{33} = [34 + 2\alpha, 46 - 5\alpha].$$

Thus, the fuzzy optimal solution for the second objective interval transportation problem is

$$\tilde{x}_{11} = (16,20,22,25);\ \tilde{x}_{13} = (14,14,16,19)\ ;\ \tilde{x}_{21} = (4,4,6,7)$$

$$\tilde{x}_{24} = (14,16,18,18); \tilde{x}_{32} = (26,27,32,34);\ \tilde{x}_{33} = (34,36,41,46).$$

The fuzzy objective value is $\tilde{Z}^2 = (186,413,597,936)$ and $\mathrm{R}(\tilde{Z}^2) = 533$.

Now, we construct a linear compromise function of the problem and determine its optimal solution and their corresponding efficient solution

$$\tilde{C}^L = \frac{1}{1054}\tilde{C}^1 + \frac{1}{533}\tilde{C}^2$$

The optimal solution for the single objective problem given is

$$\tilde{x}_{11} = (20,20,23,28);\ \tilde{x}_{13} = (10,14,15,19)\ ;\ \tilde{x}_{21} = (20,22,22,23)$$

$$\tilde{x}_{24} = (26,27,32,34)\ ; \tilde{x}_{32} = (16,20,22,25);\ \tilde{x}_{33} = (16,18,19,21)$$

Where $\tilde{Z}^1 = (596,865,1189,1566);\ \tilde{Z}^2 = (1432,1823,2250,2843\ )$
The adjacent extreme points to optimal solution are given in Table 1.

Table 1: Adjacent extreme points to optimal solution

Number	Solution of FMOTP
1	$x_{11} = (20,20,23,28); x_{12} = (10,14,15,16); x_{21} = (20,22,22,23);$ $x_{24} = (26,27,32,34); x_{32} = (6,6,7,9)\ ;\ x_{33} = (28,30,34,37)$
2	$x_{13} = (10,14,1516); x_{12} = (20,20,23,28); x_{21} = (40,43,45,50);$ $x_{24} = (6,7,8,9); x_{32} = (16,20,22,25)\ ;\ x_{33} = (16,18,19,21)$
3	$x_{11} = (30,34,38,44); x_{21} = (6,7,9,10); x_{21} = (10,14,15,16);$ $x_{24} = (26,27,32,34); x_{32} = (6,6,7,9)\ ;\ x_{33} = (28,30,34,37)$
4	$x_{11} = (30,34,38,44); x_{21} = (6,7,9,10); x_{23} = (10,14,15,16);$ $x_{24} = (26,27,32,34); x_{32} = (16,20,22,25)\ ;\ x_{33} = (16,18,19,21)$
5	$x_{11} = (2,4,4,7); x_{13} = (28,30,34,37); x_{21} = (20,22,22,23);$ $x_{24} = (26,27,32,34); x_{31} = (16,18,19,21)\ ;\ x_{32} = (16,20,22,25)$
6	$x_{11} = (2,4,4,7); x_{13} = (28,30,34,37); x_{21} = (38,39,41,43);$ $x_{24} = (8,11,13,13); x_{32} = (16,20,22,25)\ ;\ x_{34} = (16,18,19,21)$

Table 2 shows efficient solutions from which, decision maker can select the preferred solution to his fuzzy multi-objective transportation problem.

Table 2: Efficient solutions

Number	Solution of FMOTP	Multi-objective value of FMOTP
1	$\tilde{x}_{11} = (20,20,23,28);\ \tilde{x}_{13} = (10,14,15,16);$ $\tilde{x}_{21} = (20,22,22,23); \tilde{x}_{24} = (26,27,32,34)\ ;$ $\tilde{x}_{32} = (16,20,22,25);\ \tilde{x}_{33} = (16,18,19,21)$	$\tilde{Z}^1 = (596,865,1189,1566)$ $\tilde{Z}^2 = (1432,1823,2250,2843)$
2	$\tilde{x}_{11} = (16,20,22,25);\ \tilde{x}_{13} = (14,14,16,19);$ $\tilde{x}_{21} = (4,4,6,7)\tilde{x}_{24} = (14,16,18,18)\ ;$ $\tilde{x}_{32} = (26,27,32,34);\ \tilde{x}_{33} = (34,36,41,46)$	$\tilde{Z}^1 = (3260,3831,4656,5400)$ $\tilde{Z}^2 = (186,413,597,936)$

3	$\tilde{x}_{11} = (20,20,23,28)$; $\tilde{x}_{13} = (10,14,15,19)$; $\tilde{x}_{21} = (20,22,22,23)$; $\tilde{x}_{24} = (26,27,32,34)$; $\tilde{x}_{32} = (16,20,22,25)$; $\tilde{x}_{33} = (16,18,19,21)$	$\tilde{Z}^1 = (596,865,1189,1566)$ $\tilde{Z}^2 = (1432,1823,2250,2843)$
4	$\tilde{x}_{11} = (20,20,23,28)$; $\tilde{x}_{12} = (10,14,15,16)$; $\tilde{x}_{21} = (20,22,22,23)$; $\tilde{x}_{24} = (26,27,32,34)$; $\tilde{x}_{32} = (6,6,7,9)$; $\tilde{x}_{33} = (28,30,34,37)$	$\tilde{Z}^1 = (1004,1379,1827,2174)$ $\tilde{Z}^2 = (1128,1369,1755,2347)$
5	$\tilde{x}_{13} = (10,14,1516)$; $\tilde{x}_{14} = (20,20,23,28)$; $\tilde{x}_{21} = (40,43,45,50)$; $\tilde{x}_{24} = (6,7,8,9)$; $\tilde{x}_{32} = (16,20,22,25)$; $\tilde{x}_{33} = (16,18,19,21)$	$\tilde{Z}^1 = (756,1032,1323,1739)$ $\tilde{Z}^2 = (1412,1834,2295,2917)$
6	$\tilde{x}_{11} = (30,34,38,44)$; $\tilde{x}_{21} = (6,7,9,10)$; $\tilde{x}_{22} = (10,14,15,16)$; $\tilde{x}_{24} = (26,27,32,34)$; $\tilde{x}_{32} = (6,6,7,9)$; $\tilde{x}_{33} = (28,30,34,37)$	$\tilde{Z}^1 = (680,966,1432,1804)$ $\tilde{Z}^2 = (1468,1904,2368,3003)$
7	$\tilde{x}_{11} = (30,34,38,44)$; $\tilde{x}_{21} = (6,7,9,10)$; $\tilde{x}_{23} = (10,14,15,16)$; $\tilde{x}_{24} = (26,27,32,34)$; $\tilde{x}_{32} = (16,20,22,25)$; $\tilde{x}_{33} = (16,18,19,21)$	$\tilde{Z}^1 = (912,1362,1792,2220)$ $\tilde{Z}^2 = (1502,2008,2473,3083)$
8	$\tilde{x}_{11} = (2,4,4,7)$; $\tilde{x}_{13} = (28,30,34,37)$; $\tilde{x}_{21} = (20,22,22,23)$; $\tilde{x}_{24} = (26,27,32,34)$; $\tilde{x}_{31} = (16,18,19,21)$; $\tilde{x}_{32} = (16,20,22,25)$	$\tilde{Z}^1 = (1348,1679,2044,2553)$ $\tilde{Z}^2 = (976,1379,1684,2213)$
9	$\tilde{x}_{11} = (2,4,4,7)$; $\tilde{x}_{13} = (28,30,34,37)$; $\tilde{x}_{21} = (38,39,41,43)$; $\tilde{x}_{24} = (8,11,13,13)$; $\tilde{x}_{32} = (16,20,22,25)$; $\tilde{x}_{34} = (16,18,19,21)$	$\tilde{Z}^1 = (844,1084,1436,1892)$ $\tilde{Z}^2 = (1236,1658,2060,2617)$

If decision maker selects the preferred solution to fuzzy multi-objective transportation problem using the fuzzy efficient solutions of the previous steps, the algorithm ends. Otherwise, he must find the adjacent extreme points with the fuzzy efficient solutions and add fuzzy efficient solutions corresponding to the adjacent extreme points to fuzzy efficient solutions in order to obtain satisfactory results.

CONCLUSION

One of the important economic aspects of multi-objective transportation problem, is the determination of efficient distributions for a given commodity between source and destination. These efficient distributions enable the decision maker to select an appropriate solution from which the preferred solution can be determined. Often, in multi-objective transportation problems, the coefficients of the objective functions, supply and demand are fuzzy data that are determined by the decision maker. The present paper proposes a procedure for fuzzy multi-objective transportation problem where the cost coefficients of the objective functions are expressed as fuzzy numbers by the decision maker. This problem is then converted into multi-objective interval transportation problem by using α-cut set of fuzzy numbers. In the end, efficient solutions are obtained by the proposed method from which the decision maker can choose the best desirable solution among the efficient ones.

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