

THE ANALYSIS AND THE STUDY OF STUDENTS' UNDERSTANDING OF THE CONCEPT OF LIMIT BY EMPHASIZING THE CONTROL OF SCHOENFELD ACCORDING TO THE APOS THEORY

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ABSTRACT

In the Iranian educational system, Mathematics has a high position in the school curriculum. However, there are barriers to teaching and learning Mathematics, the recognition and resolve of whom has been and is one of the main concerns of math educators. The concept under study in this research is the concept of Limit, since the students have difficulty in understanding it. The aim of the present study is to analyze and study the students' understanding of the concept of Limit by emphasizing the control of Schoenfeld according to the APOS theory. The study sample is 50 male students of the eleventh grade of Mathematics and Physics in Bandar Anzali city in the academic year of 2018-19, who were randomly selected by cluster sampling method. The test tool is a questionnaire consisting of 6 questions designed by the researcher. The reliability of the test is attested by Cronbach's alpha calculation of 0.793 for the pretest and 0.812 for the posttest. The results of the study showed that the control factor is effective in increasing students' understanding of the subject of Limit based on APOS theory. On this basis, the greatest effect of the Schoenfeld control factor is on the schema and object levels, respectively. The Schoenfeld control factor had little effect on the next two levels, meaning practice and process. However, the change in the process level is slightly greater than the practice level.

Keywords: Control, Schoenfeld, Limit, APOS Theory

INTRODUCTION

In Mathematics, the concept of Limit is used to express the behavior of a function and to examine this behavior at points on the plane or at infinity. Limit is used in differential and integral calculus as well as in mathematical analysis to define derivatives as well as the concept of continuity. One of the most important concepts in Mathematics, which has always been difficult for students to understand, is the concept of Limit. Due to the connection of this concept to many other concepts, including infinitely large and infinitesimally small, continuity, derivative and integral, its correct understanding and comprehension is of particular importance, and this has led to its teaching and learning receive attention from math educators. Although this concept has been studied many times in educational research by researchers, there are still problems in understanding it by students. Some researchers, including Cornu, argue that the problems that students have in understanding comprehension of Limit are often the result of students' misconceptions about this concept, and others recognize these problems as caused by incorrect teaching methods. (Reyhani et al. 2018).

There are several methods to identify created problems in understanding concepts, including the concept of Limit. One of these methods is to study how concepts and structures, which students use to learn concepts in their minds, are formed. Investigating the mental structures of learners at any level can help math educators design concepts so that they lead to the growth and development of concepts in the minds of students. Learning theories can be used to examine the structures formed in the mind. The goal of any learning theory is to improve teaching and learning experiences in order to help learners progress in the learning process. (Wever, 2010).

Schoenfeld method for teaching Mathematics is that it includes not only the application of problem-solving techniques, but also reasoning, problem management, and the use of control approaches and strategies. In the classroom, Schoenfeld placed students in small groups to solve problems, and during the solving process, he gave himself the necessary guidance as a counselor to the students. The grouping of students made it possible for Schoenfeld to have more supervision over how they worked, and in addition, the students' ability to choose a solution and discuss it with each other developed.

APOS theory is a theory of constructivism which pays attention to how the concept of Mathematics is constructed in the learner's mind. In fact, this theory expresses a cycle of construction of concepts which occurs in the learner's mind when learning a mathematical concept. APOS theory was proposed by Dubinsky in 1991 based on one of Piaget's theories and in order to reconstruct it in the field of academic Mathematics (Asiala et al., 1997). This theory is based on the premise that one does not learn mathematical concepts directly, and rather uses mathematical structures to give meaning to a mathematical concept. It is easy for a person to learn mathematics if they have the right mental structure for a concept, however, it is almost impossible to learn the concept if they do not have the right mental structures. The structures discussed in the APOS theory include practice, process, object, and schema. In other words, APOS theory begins with practices, moves through processes, and after turning them into objects reaches to the schema (Tal, 1999).

Given the experience of teaching in different grades and disciplines in Mathematical courses, one of the topics that has been and continues to be a constant concern for the researcher has been the subject of Limit. Because many students have trouble with regard to this concept. The basis of their worldview is that Mathematics is a set of complex formulas and calculations that are not related to the world of reality and everyday activities, so they are not tangible and understandable, and only certain people have the ability to understand them.

Rafael Marrtinez-Planell and Maria Trigueros, in 2019, conducted a study entitled "Using Research Cycles in APOS: A Case of Two-Variable Functions." This paper is a study based on the APOS theory, which includes a three-cycle study of students' learning of the main idea of two-variable functions and its graphical representation. Each of the three research cycles used semi-structured interviews with students to predict, with regard to the mental structures (genetic analysis), whether students might develop classroom support activities based on interview results and use continuous guess improvement, in order to understand the functions of the two variables. For the first time, this paper has summarized the findings of the literature review from the first two research periods and gathered the results of the third and final cycle. The final results show that students who had special activities based on the findings of the first period of research were more likely to exhibit behavior consistent with understanding the



process of performance of the two variables. An important contribution of the paper is that it shows how different APOS research cycles can be used to improve students' understanding of a mathematical concept. Also, the descriptions of the three research courses provide a potentially useful guidance for improving students' learning of the performance of two variables.

Isabel Garcia-Martinez and Marcela Parraguez, in 2017, conducted a study entitled "The Basic Step in Making the Principle of Mathematical Induction Based on APOS Theory." Using the APOS theory as a framework and a case study from the perspective of methodological design of the APOS theory, this study presents a model of the Principle of Mathematical Induction (PMI) in higher education. Based on the existing evidence in the university classrooms and the results of the initial assessment, the genetic analysis designed by Dubinsky and Levine for this concept was reconstructed and introduced and defined the basic step in the principle of mathematical induction as a mental process. Using this modified genetic analysis, the products of four university students are analyzed to support or reject the proposed structures. The results show that the modified genetic analysis is sustainable, and the inclusion of the basic step as a mental process in the observed cognitive model of the principle of mathematical induction is to be observed by students. The used tools provide activities related to the teaching sequence of the principle of mathematical induction at the university level. This case study on four students by Martinez and Parraguez showed that using the APOS theory as a research framework leads to the creation and production of activities which are needed for the continuity of education in academic Mathematics. Because the obtained information is based on evidence which show that the mental processes of students are predictable and controllable.

Reyhani and Sharifi (2018) in a study examined students' understanding of the concept of Limit based on APOS theory. The results of this study showed that most students do not have a clear understanding of the concept of Limit and solve Limit problems correctly if they have access to a routine way to solve it. The study also found that constructions of the concept of Limit were very imperfect in the minds of most students, and that these weak structures not only affected their understanding of the concept of Limit, but also affected their understanding of related concepts, including continuity.

In this study, in order to examine students' understanding of the concept of Limit, we first identify the necessary structures for understanding the concept of Limit in the form of APOS theory levels. Then, by examining the structures created in the minds of students, we will assess their understanding of the concept of Limit. This study will help us to relatively identify some of the problems that have arisen in this area.

MATERIALS AND METHODS

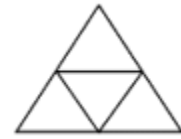
The research method in this study is quasi-experimental (quasi-empirical) method which is done by considering the experimental group and the evidence of this research. The studied population in this study includes 11th grade male students of Mathematics and Physics in Bandar Anzali city of Guilan province, in the academic year of 2018-19, which included 179 people. The statistical sample of this study, including 50 people from 2 high schools, was randomly selected.

Research Executive Process



After conducting a pre-test of the two control and experimental groups, which included 6 questions on the subject of Limit in Calculus 1 book, six sessions of one-hour extracurricular class were held for the experimental group. In these sessions, students' control skills were strengthened. These skills were accompanied with practice and repetition, and targeted problem solving, including Schoenfeld's perspectives of control steps, including analysis, design, exploration, execution, and review, so that students are put into practice and make appropriate strategic decisions. To solve the problem. To avoid the effect of other variables in this study, examples of the subject of Limit were avoided. In these sessions, the questions were reviewed and solved in the form of group activities, and the teacher played a guidance role and counselor, and while solving the problems, he or she constantly emphasized on the control recommendations. Here are some examples of issues that were discussed in extracurricular activities for students:

- 1- In the equilateral triangular triangle according to the figure, we have connected the middle of the edges to each other. What is the ratio of the surface area of all the triangles to the surface area of all the trapezoids?



- 2- How many zeros will the number $1 \times 2 \times 3 \times \dots \times 100$ end up with?
- 3- Four branches of tulips and five branches of carnations cost less than 22000 and six branches of tulips and three branches of carnations cost more than 24000. Are two tulips more valuable or three carnations?

In this study, the data were collected using a descriptive test, which was made by the researcher. All questions are at the level of knowledge and information of students with regard to the concept of Limit. In designing the test questions, the opinions and experiences of the respected professors of Mathematics education were used.

The Content Validity Ratio (CVR) index was used to determine the content validity of the test questions.

In the current study, pre-test and post-test questions were designed based on the contents of the textbook of Calculus 1, as well as the consultation and consensus of several math teachers, each with 20 questions. Then, two separate questions were given to 16 experienced math teachers along with a narrative determination questionnaire, and after reviewing their comments and answers, 6 questions of the designed questions which had the highest acceptable CVR were selected for the pre-test, and 6 questions with the highest acceptable CVR were selected for the post-test.

To determine the reliability of the pre-test, this test was performed on 20 students outside the statistical sample and after performing the test on them, the correction of the papers was done in 5 descriptive categories: excellent, good, average, weak and very weak. By assigning the numbers 1 to 5, the answers were categorized as Likert and then quantified. The same procedure was repeated for post-test. With the help of spss software, the Cronbach's alpha rate

was calculated for both tests, which in the pre-test was obtained as 0.793, and in the post-test as 0.812, which indicates an acceptable reliability of both tests.

RESULTS AND FINDINGS:

In this research, many distribution tables based on absolute frequency have been used to display the results of the work. This type of statistics describes society and aims to calculate the parameters of society. The data were analyzed using descriptive statistics. An example of this analysis is as follows:

Descriptive Analysis of the First Post-test Question

1- Consider the following functions and answer the questions:

$$f(x) = 2x + 1 \quad , \quad g(x) = 2x + 1 \quad (x \neq 2) \quad , \quad h(x) = \begin{cases} x + 2 & x \neq 2 \\ 3 & x = 2 \end{cases}$$

A) Obtain the following values if defined:

$$f(2) = \dots \quad g(2) = \dots \quad h(2) = \dots$$

B) Calculate the following Limits:

$$\lim_{x \rightarrow 2} f(x) = \dots \quad \lim_{x \rightarrow 2} g(x) = \dots \quad \lim_{x \rightarrow 2} h(x) = \dots$$

The frequency distribution of students' answers to the first question's Part A is given in Table 1.



Table 1 - Table for the Frequency of Correct Answers in Part A of Question 1

	$f(2)$			$g(2)$			$h(2)$		
	Experiment	Control	Total	Experiment	Control	Total	Experiment	Control	Total
Number of Correct Answers	19	19	38	16	16	32	17	17	33
Total Number of Students	25	25	50	25	25	50	25	25	50

Part A of the first question consists of three parts, and the performance of students in answering each of them is different. 76% of students in both groups scored $f(2)$ correctly. 64% got the right amount of $g(2)$ and 66% got the right amount of $h(2)$.

The frequency distribution of students' answers to the first question's Part B is given in Table 2.

Table 2 - Table for the Frequency of Correct Answers in Part B of Question 1

	$\lim_{x \rightarrow 2} f(x)$			$\lim_{x \rightarrow 2} g(x)$			$\lim_{x \rightarrow 2} h(x)$		
	Experiment	Control	Total	Experiment	Control	Total	Experiment	Control	Total
Number	18	17	35	16	16	32	17	17	34

of Correct Answers									
Total Number of Students	25	25	50	25	25	50	25	25	50

The results show that 70% of students correctly calculated $\lim_{x \rightarrow 2} f(x)$. In the calculation of $\lim_{x \rightarrow 2} g(x)$, the correct answers were 64% and in the calculation of $\lim_{x \rightarrow 2} h(x)$, the correct answers were 68%.

Here are some examples of students' answers to the first question brought in the following figures:

Figure 1 Sample Answer to the First Post-test Question

Consider the following functions, and answer the questions:

$$F(x) = 2x + 1, g(x) = 2x + 1 (x \neq 2), h(x) = \begin{cases} x = 2 & x \neq 2 \\ 3 & x = 2 \end{cases}$$

A) Obtain the following values, if they are defined:

$$F(2) = 6 \quad g(2) = 5 \quad h(2) = 3$$

B) Calculate the following Limits:

$$\lim_{x \rightarrow 2} f(x) = 6 \quad \lim_{x \rightarrow 2} g(x) = 5 \quad \lim_{x \rightarrow 2} h(x) = 4$$

Consider the following functions, and answer the questions:

$$F(x) = 2x + 1, g(x) = 2x + 1 (x \neq 2), h(x) = \begin{cases} x = 2 & x \neq 2 \\ 3 & x = 2 \end{cases}$$

A) Obtain the following values, if they are defined:

$$F(2) = 5 \quad g(2) = \text{Doesn't exist} \quad h(2) = 3$$

B) Calculate the following Limits:

$$\lim_{x \rightarrow 2} f(x) = 5 \quad \lim_{x \rightarrow 2} g(x) = \text{Doesn't exist} \quad \lim_{x \rightarrow 2} h(x) = 4$$

Comparing the percentage of correct answers in pre-test and post-test in the first question does not indicate a significant difference. In other words, the control factor did not affect the correct response rate of students in this question.

By examining the students' scores of the experimental group and the control group in the pre- and post-test scores, statistical indicators were obtained, and according to the information, it is observed that the mean score of students in experimental group has increased in post-test

compared to the pre-test by 3.16 under the influence of Schoenfeld control factor. The graph of the mean scores of students in the two groups of control and experimental groups is given in Figure 1:

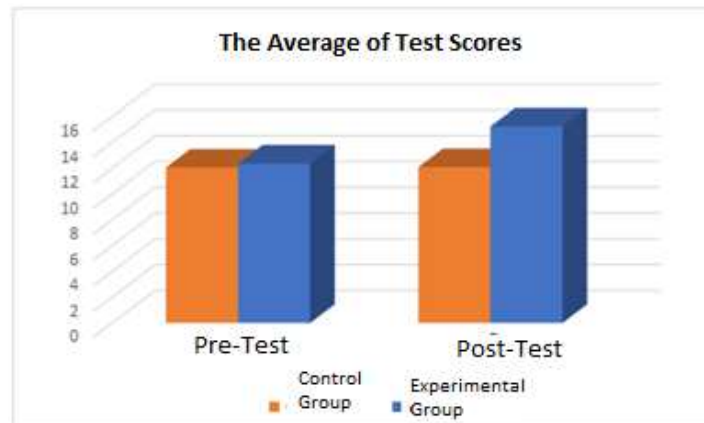


Diagram 1: Column Diagram of Students' Mean Grades

Statistical Analysis by Inferential Statistics

As can be seen in Table 3, the Kolmogorov-Smirnov test is not significant for the score ($p = 0.0555$) and therefore, the student's score has a normal distribution.

Table 3: Kolmogorov-Smirnov Test

p	Z Kolmogorov-Smirnov	Standard Deviation	Mean
.0555	.793	3.31	12.8

Table 4: The output of the SPSS software is related to the Levin test results. As can be seen, the value of Sig, which is the same p-value, is greater than the error level of $\alpha = 0.05$ of the test, therefore the assumption that the variance of the two communities is equal is not rejected.

Table 4: Levin Test Result

	Levene's Test for Equality of Variances	
	F	Sig.
Equal variances assumed	1.349	.253
Equal variances not assumed		

Then, the T-tests, which analyze the tests for the mean equality of the two communities, were examined, some examples of which are as follows:

Table 5: Comparison of the Frequency of Correct Answers, Before and After the Intervention by the Experimental and Control Groups (Question 1)

		Experiment	Control	T-Test
Question1 – A	Before	(72%) 18	(68%) 17	P= .619
	After	(76%) 19	(76%) 19	P= .102

		Experiment	Control	T-Test
Question 1 – B	Before	(56%) 14	(56%) 14	P= .788
	After	(64%) 16	(64%) 16	P= .106
		Experiment	Control	T-Test
Question1 – C	Before	(64%) 16	(68%) 17	P= .920
	After	(68%) 17	(64%) 16	P= .637
		Control	Experiment	T-Test
Question 1 – D	Before	(68%) 17	(68%) 17	P= .847
	After	(72%) 18	(68%) 17	P= .149
		Experiment	Control	T-Test
Question1 – E	Before	(68%) 17	(60%) 15	P= .457
	After	(64%) 16	(64%) 16	P= .720
		Experiment	Control	T-Test
Question 1 – F	Before	(60%) 15	(64%) 16	P= .129
	After	(68%) 17	(68%) 17	P= .455

According to the test results in Table 5, both before and after the intervention of control, the difference between the two control and experimental groups is not significant in any question. In other words, there was not much change in this question, which was at the levels of practice and process.

Table 6: Comparison of the Frequency of Correct Answers, Before and After the Intervention by Experimental and Control Group (Question 4)

		Experiment	Control	T-Test
Question 4 – A	Before	(48%) 12	(52%) 13	P= .502
	After	(60%) 15	(52%) 13	P= .485
		Experiment	Control	T-Test
Question 4 – B	Before	(40%) 10	(52%) 13	P= .483
	After	(60%) 15	(52%) 13	P= .755
		Experiment	Control	T-Test
Question 4 – C	Before	(36%) 9	(32%) 8	P= .699
	After			
		(48%) 12	(40%) 10	P= .234
		Experiment	Control	T-Test
Question 4 – D	Before	(20%) 5	(24%) 6	P= .794
	After			
		(44%) 11	(24%) 6	P= .170

According to the test results in Table 6 in parts A and B, the percentage of correct answers after the intervention of control in the experimental group has increased significantly. Also, before the intervention of control, the difference in the percentage of correct answers between the two groups of control and experiments was not significant in either of parts A and B. In part C, both experimental and control groups had a reduction in the percentage of correct answers. There is no significant difference between the two groups in this part. In part D, the difference between the control and experimental groups after the intervention of control is significant. Considering the different levels of the parts of this question, the schema level has the highest percentage of correct answers in the experimental group. The surface of the object also differs significantly, but the level of the process does not change.

Statistical Analysis Based on APOS Framework

In analyzing the data in this study, we used the APOS framework. With this framework, it is possible to categorize students' level of understanding of different concepts. There are four levels in this classification, namely: practice, process, object, and schema. Examining and comparing Tables 7 and 8 shows that the percentage of correct answers of the students in the experimental group in most of the questions has a significant increase compared to the control group in the post-test, which confirms the effect of the control factor from Schoenfeld's perspective in increasing the understanding of students with regard to the concept of Limit.

Table 7 shows the correct answers of the experimental group students in the pre-test. According to the table, 69% of students in the experimental group understand the concept of Limit at the practice level. 57% of these students understand it at the process level, 52% at the object level, and only 24% at the schema level.



Table 7: Percentage of Correct Answers of the Experimental Group in the Pre-test Based on APOS Levels

Number of Question		Practice Level	Process Level	Object Level	Schema Level
Question 1	A	72%	-	-	-
		-	56%	-	-
		-	64%	-	-
	B	68%	-	-	-
		68%	-	-	-
		-	60%	-	-
Question 2		-	80%	-	-
		-	-	72%	-
		-	-	64%	-
Question 3	A	-	-	48%	-
	B	-	-	32%	-
		-	-	40%	-
Question	A	-	-	48%	-

4	B	-	-	40%	-
	C	-	36%	-	-
		-	36%	-	-
	D	-	-	-	20%
Question 5		-	-	-	24%
Question 6	-	-	64%	-	-
	-	-	-	80%	-
	-	-	-	40%	-
	-	-	-	-	28%

Table 8 displays the percentage of correct answers of the control group students in pre-test. According to this table, 65% of students in the experimental group understand the concept of Limit at the practice level. 59% of these students understand it at the process level, 52% at the object level, and only 25% at the schema level.

Table 8: Percentage of Correct Answers of the Control Group in the Pre-test Based on APOS Levels

Number Of Question		Practice Level	Process Level	Object Level	Schema Level
Question 1	A	68%	-	-	-
		-	56%	-	-
		-	68%	-	-
	B	68%	-	-	-
		60%	-	-	-
Question 2	-	-	88%	-	-
	-	-	-	64%	-
	-	-	-	68%	-
Question 3	A	-	-	40%	-
	B	-	-	28%	-
		-	-	-	36%
Question 4	A	-	-	52%	-
	B	-	-	52%	-
	C	-	32%	-	-
		-	32%	-	-
D	-	-	-	24%	
Question 5		-	-	-	24%
Question 6		-	72%	-	-

	~	~	80%	~
	~	~	44%	~
	~	~	~	28%

Diagram 2 displays the percentage of the correct answers of the students in two experimental and control groups in the pre-test and post-test scores, respectively, based on the APOS levels. Based on this diagram, we observe that the highest effect of the Schoenfeld control factor on the correct answers of students, which shows their understanding of the concept of Limit, is at the schema level. So that in the pre-test, the control group answered 1% more correctly to the questions of this category than the experimental group. But in the post-test, the experimental group responded 24 percent more correctly than the control group. After the schema level, the greatest effect of the Schoenfeld control factor is on the object surface. So that in the pre-test, the students of both groups were at the same level. But in the post-test, the level of students in the experimental group was 9 percent higher than students in the control group. The Schoenfeld control factor had little effect on the next two levels, practice and process. However, the change in the process level is slightly greater than the practice level.

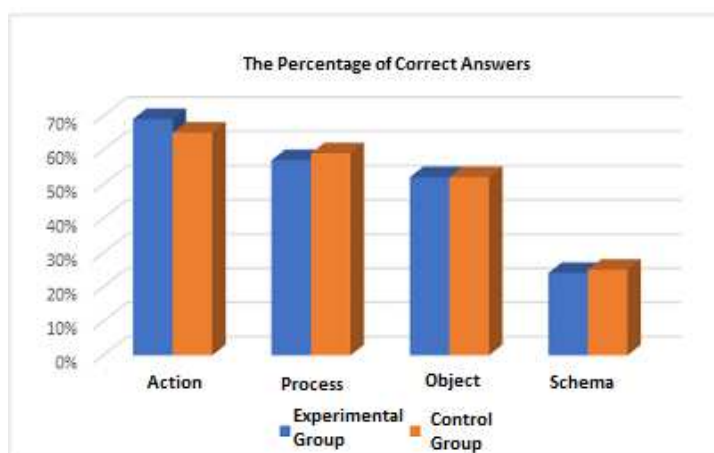


Diagram 2: Graph of the Percentage of Correct Answers of Students in the Pre-test Based on APOS Levels

DISCUSSION AND CONCLUSION

Limit is one of the most basic concepts of differential and integral calculus. In Mathematics, the subject of Limit is used to express the behaviors of a function. It also examines this behavior on points on the plate or at infinity. Limit is used in differential and integral calculus as well as in mathematical analysis to define derivatives as well as the concept of continuity. Due to the students' problems in misunderstanding of this concept, the researchers attempted, in this research, to analyze the students' understanding of the concept of Limit based on the APOS theory.

We examined the data through three methods of descriptive statistics, inferential statistics, and the APOS framework. In the descriptive statistics section, after calculating the central and dispersion indicators, we concluded that most students did not perform well in answering



conceptual questions, but performed well in answering routine questions. Also, the performance of the students in the control group was slightly more acceptable than in the experimental group, but there was no significant difference. But with the intervention of the control factor, from Schoenfeld's perspective, the performance of the experimental group in the post-test improved, and in the framework of the APOS theory we saw that this better performance is from higher levels to more apparent levels, respectively. In general, it can be concluded that in the normal situation, the level of understanding of most students is at the levels of practice or process, and in most of them, the growth and development of the concept of Limit has stopped. But Schoenfeld's control recommendations improve the students' understanding of schema, object, and, to some extent, process levels, respectively. But it has little effect on the level of the process.

Question: At what level are students' understanding of the subject of Limit within the framework of APOS theory?

In response to this question, it can be said that students are not able to be flexible between different representations of the concept of Limit of functions and most of them have understood the Limit at the level of practice, some of them have understood it at the level of process, or are capable of forming the object of Limit, and very few students have reached the correct schemas of this concept. The results of Diagram 2 and Tables 7 and 8 indicate that although in both experimental and control groups, about 3.2 of the students were at the practice level of understanding the limit, almost half of the students were at the process and object levels and only 1.4 of the students were at the schema level of understanding the concept of Limit.

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