2528-9705

Örgütsel Davranış Araştırmaları Dergisi

Journal Of Organizational Behavior Research

Cilt / Vol.: 5, Sayı / Is.: S2, Yıl/Year: 2020, Kod/ID: 71S2581

DETERMINING THE EFFECT OF CONTROLLING FACTOR ON REDUCING STUDENTS' MISUNDERSTANDING OF GEOMETRY IN GRADE 10 OF MATHEMATICS

Vahid ALAMIAN [1*], Ali ZAMANI [2], Molouk HABIBI [2]

[1] Faculty member Department of mathmatics Farhangian University, Tehran, Iran.
[2] MA, Department of mathmatics Farhangian University, Tehran, Iran.

***Corresponding author:**
Email: Vahid_alamian@yahoo.com

ABSTRACT

Students suffer from serious misunderstandings about understanding mathematical concepts, which justifies the plethora of research in mathematics education. Schoonfield's approach to mathematics education not only involves problem solving techniques but also reasoning, problem management, and the use of approaches and control strategies. This study, therefore, aimed to determine the effect of control factor on reducing students' misunderstanding of the reasoning and similarity of triangles in Grade 10 of mathematics. This was a quasi-experimental study consisting of test and control groups using pretest and posttest in both groups. The statistical population of this study included all students (n = 120) at Grade 10 of mathematics in Baneh city, in the academic year 2018-2019. The students were sampled through cluster random sampling method. A pretest was performed on two selected classes, and 20 sample students with the highest misunderstanding were selected from each class. Data were collected using two researcher-made tests. Results indicated that the controlling factor from Schoenfield's viewpoint could effectively reduce students' misunderstanding in reasoning, similarity, proportion, and polygons.

Keywords: *Students' misunderstanding, Schoenfeld method, Geometry.*

INTRODUCTION

Learning is one of the most important fields in psychology with a crucial importance in educational research therefore it has attracted the attention of many experts. Some psychology disciplines have identified their discussed issues based on learning. Learning is a relatively permanent change in behavioral ability through experience (Saif 2014). It can also be defined as making relatively persistent changes in learners' potential behavior occurring as a result of gaining experience (Gheshlaghi, 2001).

The geometry learning process in children begins before starting the school, after which they become able to express their learned concepts in formal language. Then, they can draw geometric shapes using learned drawing skills and eventually begin geometrical reasoning by learning theorems.

To endorse the importance of geometry teaching, Shahshahani considered geometry as a means of reinforcing students' imagination and creativity (Zangeneh, 1996). Geometry is also important as it is used in other branches of mathematics. Sharigin and Protasov (2004) viewed geometry as a tool for teaching problem solving strategy. Therefore, learning geometry as a main subsets of mathematics is of paramount importance due to its extent and the abilities it creates

Geliş tarihi/Recieved: 21.09.2019 – Kabul tarihi/Accepted: 16.02.2020 – Yayın tarihi/Published: 10.03.2020

in individuals. Today, geometry has been unveiled of its limited and traditional enclosure and manifested its potential strengths and extraordinary diversity, compatibility, and flexibility, which render it one of the most widespread and globally popular tools in all subsets of mathematics (Gooya and Ghadaksaz Khosrowshahi, 2008).

In recent decades, numerous studies in different countries have reported that many students have difficulty in learning geometry hence there has been much debate about the necessity of a geometry course in school mathematics and its teaching method. Some people believe that geometry should be excluded from the school mathematics curriculum and some consider reducing the geometry volume in the curriculum. According to the researchers, this tendency is more widespread among people who have trouble in understanding and learning geometry, and this was even reported among professionals in the field of mathematics.

The problem of quantitative and qualitative decline in mathematics education in the world, particularly in Iran, is a plague the effects of which will be soon visible on the shortage of needed forces in society, which necessitates research in this area and in general on teaching mathematics. Other than socioeconomic issues, one of the reasons for this decline is that we do not know "why and how to read mathematics" (Rajaie, 1987). Moreover, misunderstandings can confuse students and lead to their failure in solving problems. Sometimes misunderstandings can cause problems in future learning of students due to the interconnected nature of mathematical concepts. Accordingly, it is necessary to analyze and resolve conceptual errors of students in mathematics in order to scrutinize the causes and solutions to misunderstandings. Therefore, teachers are facing many difficulties in teaching geometric concepts and its understanding by students and the students do not achieve the desired result in learning this course`s material and gradually move away from this schoolwork.

It is extremely important to identify and correct students` errors in mathematical concepts. Teacher's awareness of students' prior knowledge and their cognitive characteristics helps him identify and examine possible errors of students and the nature and thinking of such errors.

During the years of teaching, teachers have suffered from the common mistakes of students. Although many students are highly intelligent but they may replace basic concepts with their misunderstandings and achieve wrong results. Whenever they are asked whether or not they have learned the lesson, they answer loudly "yes", while they have replaced the main concept with their notions of the subject. Consequently, appropriate strategies are needed to identify and amend these misunderstandings. This has always been in the focus of mathematics educators and researchers, leading to the creation of different theories in geometry learning. One of these solutions is to use the four-step problem-solving model presented by Schoenfeld.

Inspired by the four-step model of Polya's problem solving, Schoenfeld examined factors influencing mathematical problem solving. He considered the factors as knowledge resources, mathematical problem-solving approaches, control, and problem-solving belief systems. The preliminary results highlighted the role of these factors and particularly that of control as a determining factor. Likewise, this study aimed to examine the effect of the controlling factor on students' misunderstanding of the reasoning and similarity of triangles in Grade 10 of mathematics.

THEORETICAL BASICS OF THE RESEARCH

Geometrical problems

Geometry is not a set of definitions and formulas, rather, it is the ability to see, imagine, and think. It is, therefore, something more than merely a subject in mathematics or a curriculum in school mathematics. The following reasons are enumerated for teaching and learning geometry:

- Geometry is a phenomenon of human culture;
- Using geometry, students can develop ethics and ethical principles;
- Geometry prepares students' minds for higher education;
- Geometry develops aesthetic sense in students;
- Geometry reflects well the history of human thought.

Ven de Walle (2001) mentioned the following reasons for the importance of learning geometry: Geometry can provide a perfect understanding of the universe. It is found in the Solar System structure and even in animals, geological formations, rocks, glass, plants, and flowers. There are also geometric shapes in most of our artificial world, such as art, architecture, cars, machines, and almost in everything created by man.

- Geometric discoveries can develop problem-solving skills. Spatial reasoning is an important form of problem solving, which is in turn one of the main reasons for learning mathematics.
- Geometry plays a key role in the study of other mathematical areas. For example, the concept of deduction is related to the geometrical deductive and inductive structures. Ratio and proportion are directly correlated to the geometrical concept of similarity. There are also clear relationships between measurement and geometry.
- Geometry is used daily by many people, particularly scientists, architects, artists, and engineers, as well as at home to build a fence or design a garden.
- Geometry is enjoyable and generally enhances students' adaptation to mathematics.

One of the main features of geometry is the visualization ability, which may be a geometrical feature justifying its inclusion in the school mathematics curriculum, as almost no other lessons can be substituted for geometry (Zangeneh and Goya, 2002).
Keith Jones (2000 quoted from Sir Christopher Zeeman) considered geometry to include a branch of mathematics that applies understanding and visual insight (our most predominant sense) for recalling theorems, understanding of proof, inducing conjecture, and understanding of reality as well as providing a general insight (Reihani, 2005).
High school geometry mostly includes flat Euclidean geometry, which emphasizes logic and proof, combines formal and informal approaches to geometrical content, and relies on the subject matter and much attention to visualization and problem solving; hence, mathematics teachers' need to prepare themselves to teach this lesson. The United States Conference Board for the Mathematical Sciences (CBMS) conducted a research on the knowledge needed for mathematics teachers to prepare for teaching high school geometry. The results showed that the required knowledge is:

1. Mastering the basic concepts and principles of Euclidean geometry in plane and space.

2. Understand the nature of reasoning and the role it plays in the development of mathematics and the ease of proof.
3. Understand and be able to deal with a variety of methods, related concepts, and coordinate, vector, and transformation representations.
4. Understand trigonometry from a geometric viewpoint and being skilled in its application to problem solving.
5. Knowledge of some important geometrical topics and their applications, such as tiling, fractals, computer drawings, robotics, and visualization.
6. Ability to apply dynamic geometric drawing tools to conduct geometric research with an emphasis on visualization, pattern recognition, conjecture, and proof.

According to the above, it can be inferred that learning geometry is considered to be the main part of mathematics, and the geometrical concepts can be useful in later learning if they are correctly constructed in the students' minds in primary schools. Finally, students are able to understand the applications of geometry in their surroundings. In addition, since shapes and objects are accessible in the geometry structure, it helps students to better understand the world where they live.

Reasoning and proof

The USA National Council of Teachers of Mathematics (NCTM, 2000) introduced reasoning as one of the education process standards in school mathematics and stated that the curriculum from preschool by the end of Grade 12 should enable all students to:

- Recognize reasoning and proof as basic mathematical aspects;
- Create and examine mathematical conjectures;
- Complete and evaluate mathematical arguments and proofs;
- Choose and apply different methods of reasoning and proof.

Research has indicated that some school and university students do not understand the importance of proof and, in some cases, do not find it necessary to learn mathematical proposition proofs (Clements and Alerton, 1996).
Teaching and learning at any level, especially for students at various levels of education, is associated with misunderstanding, thus misunderstandings are expected for teaching mathematics and geometry. Mistakes of students in learning mathematics and attempt to analyze error in math instruction have a long history worldwide. Identification and elimination of students' misunderstandings about mathematical concepts is critically important. This is because if we be aware of students' prior knowledge and cognitive characteristics, we may be able to know students better and obtain information about the time and possibility of making mistakes. Although students learn concepts, they also sometimes learn conceptual errors contrary to what they are taught. Most often, patterns of error in students indicate that conceptual errors are learned by students.

Misunderstandings and their roots

According to Drew (2005), misunderstanding can result from inaccurate application of a rule or over-generalization, or another understanding of the mathematical situation. Buttel stated that "misunderstanding arises from the fact that a student does not understand or misunderstand a subject, which are not actually due to carelessness or inattention to the activity, but have deeper roots. Student's misunderstanding may stem from their past experiences and knowledge in daily life, be seriously retained by students, thus their outcomes delay learning. According to Olivear, mistakes and misunderstandings are the result of students' attempts in building their knowledge hence their occurrence is unavoidable and should not be treated as terrifying things requiring eradication. Instead, he recommended that the best manner of paying attention to misunderstandings is to use them as part of the learning process. It is, therefore, necessary to create an atmosphere in the mathematics classroom in order to have patience to students' mistakes and misunderstandings and use them as opportunities to enhance learning mathematics.

Students' conceptual errors or mathematical misunderstandings do not necessarily result from non-focus, carelessness, and so on, but are rooted in ones' mental structures or schemas. For this reason, understanding the influence of mental schemas of misunderstandings requires extensive research (Hesam, 2005). It can, therefore, be inferred that most of students' misunderstandings in mathematics result from mental schemas and their formation and development; it is often observed that the schemas created by students are not integrated and cohesive. Consequently, the origin of misunderstandings should be sought in mental schemas of individuals (Azarang, 2008).

Freudenthal (1968) identified one of the important research issues in mathematics education to examine the process of students' mathematical performance and find their mistakes, thereby identifying the inability of children in understanding mathematics. To deal with misunderstandings, Vygotski believed that a novice learner should be supervised by an expert. Piaget also agreed him in this respect and believed that student learning should be guided by the teacher.

Soltani (2012) conducted a research on the identification and causes of misunderstandings in the subject of function limits on Grade 3 of high school students in Quchan city. It was concluded that students had multiple misunderstandings about the concept of limit and one of the reasons for the misunderstanding was identified to be the quantity and quality of students' curricula and textbooks. The author believed that the use of single terms and expressions in textbooks and the use of certain words by teachers can cause students to have a poor understanding of the concept, leading to the misunderstandings of limit concept.

Schoenfeld's problem solving process

Schoenfeld's approach to mathematics education not only involves problem solving techniques but also reasoning, problem management, and the use of approaches and control strategies. In the classroom, Schoenfeld assigned students to small groups to solve problems, and provided the students with the necessary guidance as a counselor during the solving process. Grouping the students enabled Schoenfeld to further control over their works, and also expanded students' ability to choose a solution and discuss with one another. In Schoenfeld's classroom, students initially answered three questions: 1) What (exactly) are you doing?, 2) Why are you doing it?, and 3) How does it help you reach a solution?

Asking these questions would help students reconsider their problem-solving process and make sure of their selected path. In fact, students ask themselves here whether or not to continue the path (Faramarzpour and Fadaei, 2016).

From the viewpoint of Schoenfeld (1985), factors influencing mathematical problem solving, and indeed the overall framework for analyzing problem-solving behavior, consist of four main parts:

Control is related to the use of one's available knowledge. Selected behaviors include making a plan, selecting goals and sub-goals, monitoring and evaluating solutions while making progress on them, and revising or leaving the plans when evaluations imply making such a decision.

METHODS

This research was a quasi-experimental study consisting of test and control groups using pretest and posttest in both groups. The statistical population of this study included all students (n = 120) at Grade 10 of mathematics in Baneh city, in the academic year 2018-2019. The students were sampled through cluster random sampling method. Pretest was performed on two selected classes, and 20 sample students with the highest misunderstanding were selected from each class. Data were collected using two researcher-made tests. The first and the second tests were performed to detect misunderstandings and as a posttest to examine the impact of the independent variable, respectively.

A pre-test was first held after sampling and assigning the test and control groups. The test consisted of six standardized questions out of 18 questions made up of Grade 9 mathematics and Grade 10 geometry textbooks. The validity of questions was determined using the CVR table and asking questions from experts, most of whom had a master's degree in mathematics education. Students' misunderstandings were extracted after reviewing and correcting the worksheets, and 20 students with the uppermost misunderstandings were selected from each class. To determine the reliability, a Cronbach's alpha of 0.737 indicated a good reliability. The test group received six sessions of Schoenfeld-based extracurricular training to reinforce control skills in this group. The control group received the same extracurricular class in the traditional manner with which their teachers used to teach previously. It was attempted to present issues to the other group similar to those raised in the test group, but with a different solving approach. Control skills during class exercises and activities were emphasized on the test group.

Research questions

Question 1: Does controlling factor affect students' misunderstanding about reasoning?

Question 2: Does controlling factor affect students' misunderstanding on the proportionality and similarity of triangles?

Question 3: Does controlling factor affect students' misunderstanding about polygons?

FINDINGS

A pretest was performed on students to identify their misunderstandings, resulting in the identification of five misunderstanding types as presented below.

Table 1. Classification of misunderstandings

Main category	Sub-category	Misunderstanding code	Supporting Ref.
Misunderstanding in reasoning and proof	No recognition of the equality of the corresponding components (sides and angles)	1	McCrone and Martin (2004)
	Use of a sentence to provide a reason for modular arithmetic	2	Clements and Batista (1994)
	Inability to complete or change a shape	3	Clements and Batista (1992)
Misunderstanding in proportionality and similarity	Inaccurate use of components	4	Mahlabela (2012)
	Use equality of sides rather than proportionality to prove similarity	5	Mahlabela (2012)

Table 2 summarizes the pretest results made in the test group (n = 20) to enumerate the misunderstandings.

Table 2. Distribution of misunderstandings among students in Class A (test group) in the pretest

Student code	1	2	3	4	5	Total
Total	12	9	16	6	5	92

Table 2 indicates that the highest frequency in the test group for the pretest belonged to misunderstanding No. 3 regarding the inability to make or complete changes in shapes. The least frequent misunderstanding was seen in misunderstanding No. 5 about using the similarity of sides rather than their proportionality to prove similarity.
The data obtained from the pretest in the control group (n = 20) to enumerate the misunderstandings have been summarized in Table 3.

Table 3. Distribution of misunderstandings among students in Class B (control group) in the pretest

Misunderstanding type	1	2	3	4	5	Total
Total	13	8	17	5	5	87

According to Table 2, the maximum frequency (17) in the test group for the pretest was observed in misunderstanding No. 3 regarding the inability to make or complete changes in shapes. The minimum frequency (5) of misunderstandings belonged to misunderstanding No. 5 about using the similarity of sides rather than their proportionality to prove similarity.
Table 4 represents the number of misunderstandings in the posttest for the test group (n = 20).

Table 4. Distribution of misunderstandings among students in Class A (test group) in the posttest

Misunderstanding type	1	2	3	4	5	Total
Total	4	2	5	0	2	32

Table 5. Distribution of misunderstandings among students in Class B (control group) in the posttest

Misunderstanding type	1	2	3	4	5	Total
Total	7	4	12	3	3	47

According to Table 4, the maximum frequency (5) in the test group for the pretest was observed in misunderstanding No. 3 regarding the inability to make or complete changes in shapes. The lowest frequency of misunderstandings belonged to misunderstanding No. 4 about using proportionality of components incorrectly.
According to Table 5, the maximum frequency (12) in the test group for the pretest was observed in misunderstanding No. 3 concerning the inability to make or complete changes in shapes. The lowest frequency of misunderstandings belonged to misunderstanding No. 4 about using proportionality of components incorrectly and the use of similarity of sides rather than their proportionality to prove similarity.

Table 6. The equality of variances in the test and control groups based on Levene's test

Misunderstandings	Levene's statistic (F)	df 1	df 2	Sig.
1	0.0	1	38	1.0
2	0.0	1	38	1.0
3	0.669	1	38	0.419
4	0.478	1	38	0.493
5	0.669	1	38	0.419

In the Levene's test, H_0 is the equality of variances in two groups. In Table 6, levels of significance were > 0.05 for all misunderstandings, which did not reject the H_0 and confirmed the equality of variances for all reported misunderstandings in the two groups.

Table 7. Examining normal distribution of data in the posttest of the test group

Normality test						
Misunderstandings	Kolmogorov-Smirnov			Shapiro-Wake		
	Statistic	df	Sig.	Statistic	df	Sig.
1	0.487	20	0.00	0.495	20	0.00
2	0.527	20	0.00	0.351	20	0.00
3	0.463	20	0.00	0.544	20	0.00
4	--	--	--	--	--	--
5	0.527	20	0.00	0.351	20	0.00

*The software removed misunderstanding No. 7 from the list because all the data are constant (the absence of misunderstanding).

In this study, the normality of the experimental data in the posttest was verified using Shapiro-Wake and Kolmogorov-Smirnov tests, where H_0 and H_1 represented normal and non-normal distribution of data, respectively. In Table 7, the significance levels of all misunderstandings were less than 0.05 in both tests, thus rejecting the H_0 and indicating non-normal distribution of data at 0.05 error level.

Table 8. Examining normal distribution of data in the posttest of the control group

Normality test						
Misunderstandings	Kolmogorov-Smirnov			Shapiro-Wake		
	Statistic	df	Sig.	Statistic	df	Sig.
1	0.413	20	0.00	0.608	20	0.00
2	0.487	20	0.00	0.495	20	0.00
3	0.387	20	0.00	0.626	20	0.00
4	0.509	20	0.00	0.433	20	0.00
5	0.538	20	0.00	0.236	20	0.00

Table 8 also shows non-normal distribution of data in the control group. Therefore, the effects of individual trainings on each group were compared using Mann-Whitney test (nonparametric t-test for independent groups).

Table 9. Significance of reductions in misunderstandings in the test group

Misunderstandings / Indices	1	2	3	4	5
Mann-Whitney statistic	110.0	140.0	90.0	140.0	150.0
Sig. (two tails)	0.004	0.031	0.001	0.009	0.062
Sig. (one tail)	0.002	0.015	0.0005	0.004	0.031

Table 10. Significance of reductions in misunderstandings in the control group

Misunderstandings / Indices	1	2	3	4	5
Mann-Whitney statistic	140.0	160.0	170.0	180.0	180.0
Sig. (two tails)	0.61	0.173	0.262	0.435	0.298
Sig. (one tail)	0.31	0.081	0.131	0.22	0.15

Question 1: Does the controlling factor affect students' misunderstanding of reasoning?

Misunderstandings with codes 1, 2, and 3 fell into this question and their descriptive and inferential analysis were as follows:

The first misunderstanding referred to the non-recognition of the equality of the corresponding components (side and angle), particularly when two triangles share a side or an angle. The numbers of test and control students (n = 20) with this misunderstanding were observed as 12 and 13 in the pretest and 4 and 7 in the posttest, respectively. The misunderstanding reduction percentages were 67% and 46% in the test and the control group, respectively, indicating better control factor-based teaching than the traditional method. The results of inferential statistics (Tables 9 and 10) revealed significant reductions of misunderstandings in both groups with Sig. levels of 0.002 and 0.31 for the test and control groups, respectively, using Mann-Whitney statistic. This indicated that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and was approved in the control group. Therefore, both descriptive and inferential statistics confirmed that the controlling factor could effectively reduce the misunderstanding No. 1.

The second misunderstanding denoted the use of the sentence to provide a reason for cohesive. The numbers of students (n = 20) in the test and control groups having this misunderstanding were observed as 9 and 8 students in the pretest, and 2 and 4 in the posttest, respectively. Accordingly, the percentage of misunderstanding decreased by 78% and 50% in the test and control groups, respectively, indicating better performance of control factor-based teaching than the traditional method; however, an appropriate reduction was also recorded in the control group.

The results of inferential statistics (Tables 9 and 10) disclosed the significant reductions of misunderstandings in both groups with Sig. levels of 0.015 and 0.081 for the test and control groups, respectively, using Mann-Whitney statistic. This indicated that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and was approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 2 by the controlling factor. In the control group, the corresponding number was close to the index of 0.05, suggesting the successful effect of the traditional method to reduce this misunderstanding to some extent.

The third misunderstanding represented the inability to change or complete a shape. The numbers of students (n = 20) in the test and control groups presenting this misunderstanding were observed as 16 and 17 students in the pretest, and 5 and 12 in the posttest, respectively. Thus, the percentage of misunderstanding declined by 69% and 29% in the test and control groups, respectively, showing better performance of control factor-based teaching than the traditional method.

The results of inferential statistics (Tables 9 and 10) unveiled the significant reductions of misunderstandings in both groups with Sig. levels of 0.0005 and 0.131 for the test and control groups, respectively, using Mann-Whitney statistic. This indicated that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 6 by the controlling factor. As a conclusion about Schunfield's view of control function to reduce the misunderstandings, the first question proved the effectiveness of this method for reducing misunderstanding in the reasoning subject matter as it was confirmed in all three misunderstandings related to this question.

Question 2: Does the controlling factor affect students' misunderstanding of the proportionality and similarity of triangles?

Misunderstandings with codes 4 and 5 fell into this question, and their descriptive and inferential analyses were as follows:

In the fourth misunderstanding representing inaccurate use of the proportionality of components, the numbers of test and control students (n = 20) with this misunderstanding were observed as 6 and 5 in the pretest and 0 and 3 in the posttest, respectively. Hence, the percentage of misunderstanding dropped by 100% and 40% in the test and control groups, respectively, suggesting better performance of control factor-based teaching than the traditional method.
The results of inferential statistics (Tables 9 and 10) revealed significant reductions of misunderstandings in both groups with Sig. levels of 0.004 and 0.22 for the test and control groups, respectively, using Mann-Whitney statistic. This indicated that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 4 by the controlling factor.
In the fifth misunderstanding indicating the use of equality of sides instead of their proportionality for similarity proof, the numbers of test and control students (n = 20) with this misunderstanding were observed as 5 and 5 in the pretest and 2 and 3 in the posttest, respectively. Therefore, the percentages of misunderstanding dropped by 60% and 40% in the test and control groups, respectively, signifying better performance of control factor-based teaching than the traditional method.
The results of inferential statistics (Tables 9 and 10) demonstrated significant reductions of misunderstandings in both groups with Sig. levels of 0.031 and 0.15 for the test and control groups, respectively, using Mann-Whitney statistic. This indicated that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 5 by the controlling factor.
As a conclusion about Schunfield's view of control function to reduce misunderstandings, the second question proved the effectiveness of this method for reducing misunderstanding in the similarity and proportion subject matter as it was confirmed in the two misunderstandings related to this question.
Question 2: Does the controlling factor affect students' misunderstanding of the proportionality and similarity of triangles?
Misunderstandings with codes 7, 8 and 10 fell under this question.
In the seventh misunderstanding that pointed to incorrect use of components, the numbers of test and control students (n = 20) having this misunderstanding were observed as 5 and 6 in the pretest and 0 and 3 students in the posttest, respectively. Therefore, the percentages of misunderstanding dropped by 100% and 40% in the test and control groups, respectively, indicating better performance of control factor-based teaching than the traditional method.
The results of inferential statistics (Tables 9 and 10) showed significant reductions of misunderstandings in both groups with Sig. levels of 0.004 and 0.22 for the test and control groups, respectively, using Mann-Whitney statistic. This shows that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 4 by the controlling factor.

In the eighth misunderstanding referring to the use of equality of sides instead of their proportionality for similarity proof, the numbers of test and control students (n = 20) with this misunderstanding were observed as 5 and 5 in the pretest and 2 and 3 in the posttest, respectively. Therefore, the percentages of misunderstanding dropped by 60% and 40% in the test and control groups, respectively, suggesting better performance of control factor-based teaching than the traditional method. The results of inferential statistics (Tables 9 and 10) showed significant reductions of misunderstandings in both groups with Sig. levels of 0.031 and 0.15 for the test and control groups, respectively, using Mann-Whitney statistic. This shows that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 8 by the controlling factor. It can, therefore, be concluded that the controlling factor influenced the reduction of misunderstandings in the subject matter of the proportionality of triangles.

Question 3: Does the controlling factor affect students' misunderstanding in the subject matter of polygons?

Misunderstandings with codes 1, 2, 3, and 10 fell into this question.

In the first misunderstanding relating to defining special polygons (rhombus, square, etc.), the numbers of test and control students (n = 20) having this misunderstanding were 5 and 4 in the pretest and 2 and 2 in the posttest, respectively. Therefore, the percentages of misunderstanding decreased by 60% and 50% in the test and control groups, respectively, suggesting better performance of control factor-based teaching than the traditional method. The results of inferential statistics (Tables 9 and 10) displayed significant declines of misunderstandings in both groups with Sig. levels of 0.015 and 0.19 for the test and control groups, respectively, using Mann-Whitney statistic. This means that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 1 by the controlling factor.

Find the area of a rhombus with a length of 10 cm and a diameter of 12 cm.

$12 \times 2 = 24 \qquad 24 \times 10/2 = 120$

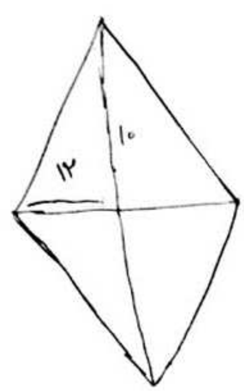

Figure 1. An example of a misunderstanding observed in the pretest

In the first misunderstanding denoting the use of polygon coordinates for subject proof, the numbers of test and control students (n = 20) having this misunderstanding were 14 and 15 in the pretest and 7 and 8 in the posttest, respectively. Therefore, the percentages of misunderstanding decreased by 50% and 47% in the test and control groups, respectively, signifying better performance of control factor-based teaching than the traditional method. The results of inferential statistics (Tables 9 and 10) represented significant declines of misunderstandings in both groups with Sig. levels of 0.014 and 0.014 for the test and control groups, respectively, using Mann-Whitney statistic. This implies that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 2 in lowering the second misunderstanding as confirmed by the controlling factor.

In the third misunderstanding, which refers to the area and environment of polygons, the numbers of students in the test and control groups (n = 20) with this misunderstanding were 11 and 8 in the pretest and 3 and 2 students in the posttest, respectively. Therefore, the percentages of misunderstanding diminished by 73% and 75% in the test and control groups, respectively, indicating better performance of control factor-based teaching than the traditional method. The results of inferential statistics (Tables 9 and 10) represented significant decreases of misunderstandings in both groups with Sig. levels of 0.004 and 0.016 for the test and control groups, respectively, using Mann-Whitney statistic. This suggests that the H_0 (no effect of Schoenfeld's control factor on the reduction of misunderstanding) was rejected in the test group and approved in the control group. Therefore, both descriptive and inferential statistics confirmed the effective reduction of misunderstanding No. 4 in lowering the second misunderstanding as confirmed by the controlling factor. Therefore, it can be inferred that the controlling factor influences the reduction of misunderstandings in polygons subject matter.

DISCUSSION AND CONCLUSION

This study mainly aimed to determine the effect of controlling factor from Schoenfeld's viewpoint on reducing students' misunderstanding in the subject matter of reasoning and similarity of triangles and polygons in Grade 0 geometry of mathematics. Necessary data for research hypotheses and questions were obtained through posttest and pretest and comparison of results. The results of Mann-Whitney statistic (Table 11) confirmed the proper performance of the teaching method based on the controlling factor from Schunfield's view. According to the Mann-Whitney test results, the significance levels of reduction in misunderstandings 1, 2, and 3 were 0.002, 0.015, and 0.0005 in the test group, respectively. This means that H_0 (no influence of controlling factor on misunderstanding reduction from Schoenfield's viewpoint) was rejected in the test group by all the misunderstandings related to the first question at a significant level of 0.05 (5%). This confirmed that the controlling factor from Schunfield's view was effective in the reduction of students' misunderstanding.

In the control group, the significance levels of reduction in misunderstandings 1, 2, and 3 were 0.31, 0.081, 0.13, which means that H_0 (no impact of controlling factor on misunderstanding reduction from Schoenfield's viewpoint) was approved by all the misunderstandings related to the first question at a significant level of 0.05. This proves that traditional education had no effects on the reduction of students' misunderstanding.

Given the above discussion in response to the first question and confirming its effect on all three misunderstandings related to this question, it can be concluded that the controlling factor from

Schoenfeld's view influenced the reduction of students' misunderstanding in the reasoning subject matter.

Table 11. Significance levels of Mann-Whitney test for misunderstandings of the first question in the two groups

Misunderstandings / Indices	1	2	3
Mann-Whitney Sig. (test group)	0.002	0.015	0.0005
Mann-Whitney Sig. (control group)	0.31	0.081	0.131

As shown in Table (12), the reduction percentages of misunderstandings 7 and 8 were 100% to 40% and 60% to 40% in the posttest of the test group in comparison with control group, indicating better performance of teaching based on the controlling factor from Schoenfeld's view in all the misunderstandings. According to the Mann-Whitney test results, the significance levels of reduction in misunderstandings 4 and 5 were 0.004 and 0.031 in the test group, respectively. This means that H_0 (no influence of controlling factor on misunderstanding reduction from Schoenfield's viewpoint) was rejected in the test group by all the misunderstandings related to the first question at a significant level of 0.05 (5%). This confirms that the controlling factor from Schunfield's view was effective in the reduction of students' misunderstanding in the subject matter of similarity and proportionality of triangles.

In the control group, the significance levels of reduction in misunderstandings 4 and 5 were 0.22 and 0.15, which means that H_0 (no impact of controlling factor on misunderstanding reduction from Schoenfield's viewpoint) was approved by all the misunderstandings related to the second question at a significant level of 0.05. This proves that traditional education had no effects on the reduction of students' misunderstanding.

Given the above discussion in response to the second question and confirming its effect on the two misunderstandings related to this question, it can be concluded that the controlling factor from Schoenfeld's view influenced the reduction of students' misunderstanding in the subject matter of similarity and proportionality of triangles.

Table 12. Significance levels of the Mann-Whitney test for misunderstandings of the second question in the two groups

Misunderstandings / Indices	7	8
Mann-Whitney Sig. (test group)	0.004	0.031
Mann-Whitney Sig. (control group)	0.22	0.15

References

Azarang, Y., (2008). Mathematics 1 and students' misunderstanding, *Roshd Journal, 26*(1), 21-16.

Clements-Ellerton, (1996). Research in Mathematics Education: Past, Present and Future. Translated by Amir Hossein Ashna, (2014). Tehran. Karafarin Publications.

Faramarz Pour, N., & Fadaei, M. R., (2016), *Control, an effective component in problem solving education from Schoenfeld's view*, Iranian Mathematical Education Conference, 6.

Gheshlaghi, M., (2001), *The Psychology of Learning*, Publisher: Mani.

Goya, Z., & Khosrowshahi Ghadaksaz, L., (2008), How do students use personally referred criteria for size estimation? *Curriculum Studies*, 10, 1-19.

Hesam, A., (2005). *A study of students' mathematical misunderstandings and the role of mental schemas in their creation*, M.Sc. thesis, Shahid Beheshti University.

Rajaee, A., (1987). How to read mathematics, *Roshd Journal* 15, 10-12.

Reihani, E., (2005). Introducing Piaget's theory of learning the geometry of mathematical education. Roshd Journal of Mathematical Education, *Educational Research and Planning Organization*, 80, 22, 12-22

Saif, A. A. (2014). *Modern educational psychology, learning, and teaching psychology*. Tehran: Doran Publishing.

Soltani, M., (2012). *Misunderstanding of Grade 3 experimental sciences and mathematics students about the concept of limit*, M.Sc. thesis, Institute of Science, Shahid Rajaee University of Teacher Education.

Ven de Walle, J., (2001), Developing Mathematical Understanding, Translated by Sepideh Chamanara, Roshd Journal of Mathematics Education, 74 (2003), 4-14, Office of Educational Aid Publications, Educational Research and Planning Organization, Ministry of Education.

Zohoori Zangeneh, B., & Gooya, Z. (2002). New perspectives on geometry education. Roshd Journal of Mathematical Education 67, 19, Office of Educational Aid Publications, Educational Research and Planning Organization, Ministry of Education.

Zohoori Zangeneh, B., (1996). Geometry Education Roundtable, First Iranian Mathematics Education Conference, Roshd Journal of Mathematical Education, 51, 22. Office of Educational Aid Publications, Educational Research and Planning Organization, Ministry of Education.